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Integrability Formulas. Part II

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Summary. In this article, we give several differentiation and integrability formulas of special and composite functions including trigonometric function, and polynomial function.

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The terminology and notation used here have been introduced in the following articles: [12], [13], [2], [3], [9], [1], [6], [11], [14], [4], [18], [7], [8], [5], [19], [10], [16], [17], and [15].

For simplicity, we use the following convention: a, x are real numbers, n is an element of $\mathbb{N}$, A is a closed-interval subset of $\mathbb{R}$, f, h, f_1, f_2 are partial functions from $\mathbb{R}$ to $\mathbb{R}$, and Z is an open subset of $\mathbb{R}$.

The following propositions are true:

(1) Suppose that
 (i) $A \subseteq Z$,
 (ii) $f = \frac{1}{(\text{the function sin})\,(\text{the function cos})}$,
 (iii) $Z \subseteq \operatorname{dom}((\text{the function ln}) \cdot (\text{the function tan}))$,
 (iv) $Z = \operatorname{dom} f$, and
 (v) f is continuous on A.

Then $\int\limits_{A} f(x)dx = ((\text{the function ln}) \cdot (\text{the function tan}))(\sup A) - ((\text{the function ln}) \cdot (\text{the function tan}))(\inf A)$.

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(2) Suppose that
(i) $A \subseteq Z$,
(ii) $f = -\frac{1}{(\text{the function sin})\ (\text{the function cos})}$,
(iii) $Z \subseteq \operatorname{dom}((\text{the function ln})\ \cdot(\text{the function cot}))$,
(iv) $Z = \operatorname{dom} f$, and
(v) f is continuous on A.

Then $\int\limits_A f(x)dx = ((\text{the function ln})\ \cdot(\text{the function cot}))(\sup A) - ((\text{the function ln})\ \cdot(\text{the function cot}))(\inf A)$.

(3) Suppose that
(i) $A \subseteq Z$,
(ii) $f = 2\,((\text{the function exp})\ \ (\text{the function sin}))$,
(iii) $Z \subseteq \operatorname{dom}((\text{the function exp})\ \ ((\text{the function sin}) - (\text{the function cos})))$,
(iv) $Z = \operatorname{dom} f$, and
(v) f is continuous on A.

Then $\int\limits_A f(x)dx = ((\text{the function exp})\ \ ((\text{the function sin}) - (\text{the function cos})))(\sup A) - ((\text{the function exp})\ \ ((\text{the function sin}) - (\text{the function cos})))(\inf A)$.

(4) Suppose that
(i) $A \subseteq Z$,
(ii) $f = 2\,((\text{the function exp})\ \ (\text{the function cos}))$,
(iii) $Z \subseteq \operatorname{dom}((\text{the function exp})\ \ ((\text{the function sin}) + (\text{the function cos})))$,
(iv) $Z = \operatorname{dom} f$, and
(v) f is continuous on A.

Then $\int\limits_A f(x)dx = ((\text{the function exp})\ \ ((\text{the function sin}) + (\text{the function cos})))(\sup A) - ((\text{the function exp})\ \ ((\text{the function sin}) + (\text{the function cos})))(\inf A)$.

(5) Suppose $A \subseteq Z = \operatorname{dom}((\text{the function cos}) - (\text{the function sin}))$ and $(\text{the function cos}) - (\text{the function sin})$ is continuous on A. Then $\int\limits_A ((\text{the function cos}) - (\text{the function sin}))(x)dx = ((\text{the function sin}) + (\text{the function cos}))(\sup A) - ((\text{the function sin}) + (\text{the function cos}))(\inf A)$.

(6) Suppose $A \subseteq Z = \operatorname{dom}((\text{the function cos}) + (\text{the function sin}))$ and $(\text{the function cos}) + (\text{the function sin})$ is continuous on A. Then $\int\limits_A ((\text{the function cos}) + (\text{the function sin}))(x)dx = ((\text{the function sin}) - (\text{the function cos}))(\sup A) - ((\text{the function sin}) - (\text{the function cos}))(\inf A)$.

(7) Suppose $Z \subseteq \mathrm{dom}((-\frac{1}{2})\,\frac{(\text{the function sin})+(\text{the function cos})}{\text{the function exp}})$. Then

(i) $(-\frac{1}{2})\,\frac{(\text{the function sin})+(\text{the function cos})}{\text{the function exp}}$ is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $((-\frac{1}{2})\,\frac{(\text{the function sin})+(\text{the function cos})}{\text{the function exp}})'_{\restriction Z}(x) = \frac{(\text{the function sin})(x)}{(\text{the function exp})(x)}$.

(8) Suppose that

(i) $A \subseteq Z$,

(ii) $f = \frac{\text{the function sin}}{\text{the function exp}}$,

(iii) $Z \subseteq \mathrm{dom}((-\frac{1}{2})\,\frac{(\text{the function sin})+(\text{the function cos})}{\text{the function exp}})$,

(iv) $Z = \mathrm{dom}\, f$, and

(v) f is continuous on A.

Then $\int\limits_{A} f(x)dx = ((-\frac{1}{2})\,\frac{(\text{the function sin})+(\text{the function cos})}{\text{the function exp}})(\sup A) - ((-\frac{1}{2})\,\frac{(\text{the function sin})+(\text{the function cos})}{\text{the function exp}})(\inf A)$.

(9) Suppose $Z \subseteq \mathrm{dom}(\frac{1}{2}\,\frac{(\text{the function sin})-(\text{the function cos})}{\text{the function exp}})$. Then

(i) $\frac{1}{2}\,\frac{(\text{the function sin})-(\text{the function cos})}{\text{the function exp}}$ is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $(\frac{1}{2}\,\frac{(\text{the function sin})-(\text{the function cos})}{\text{the function exp}})'_{\restriction Z}(x) = \frac{(\text{the function cos})(x)}{(\text{the function exp})(x)}$.

(10) Suppose that

(i) $A \subseteq Z$,

(ii) $f = \frac{\text{the function cos}}{\text{the function exp}}$,

(iii) $Z \subseteq \mathrm{dom}(\frac{1}{2}\,\frac{(\text{the function sin})-(\text{the function cos})}{\text{the function exp}})$,

(iv) $Z = \mathrm{dom}\, f$, and

(v) f is continuous on A.

Then $\int\limits_{A} f(x)dx = (\frac{1}{2}\,\frac{(\text{the function sin})-(\text{the function cos})}{\text{the function exp}})(\sup A) - (\frac{1}{2}\,\frac{(\text{the function sin})-(\text{the function cos})}{\text{the function exp}})(\inf A)$.

(11) Suppose that

(i) $A \subseteq Z$,

(ii) $f = (\text{the function exp})\ ((\text{the function sin})+(\text{the function cos}))$,

(iii) $Z \subseteq \mathrm{dom}((\text{the function exp})\ (\text{the function sin}))$,

(iv) $Z = \mathrm{dom}\, f$, and

(v) f is continuous on A.

Then $\int\limits_{A} f(x)dx = ((\text{the function exp})\ (\text{the function sin}))(\sup A) - ((\text{the function exp})\ (\text{the function sin}))(\inf A)$.

(12) Suppose that

(i) $A \subseteq Z$,

(ii) $f =$ (the function exp) ((the function cos)$-$(the function sin)),
(iii) $Z \subseteq$ dom((the function exp) (the function cos)),
(iv) $Z = \operatorname{dom} f$, and
(v) f is continuous on A.

Then $\int\limits_A f(x)dx =$ ((the function exp) (the function cos))(sup A) $-$ ((the function exp) (the function cos))(inf A).

(13) Suppose that
(i) $A \subseteq Z$,
(ii) $f_1 = \square^2$,
(iii) $f = -\frac{\frac{\text{the function sin}}{\text{the function cos}}}{f_1} + \frac{\frac{1}{\mathrm{id}_Z}}{(\text{the function cos})^{\mathbf{2}}}$,
(iv) $Z \subseteq$ dom($\frac{1}{\mathrm{id}_Z}$ (the function tan)),
(v) $Z = \operatorname{dom} f$, and
(vi) f is continuous on A.

Then $\int\limits_A f(x)dx = (\frac{1}{\mathrm{id}_Z}$ (the function tan))(sup A) $-$ ($\frac{1}{\mathrm{id}_Z}$ (the function tan))(inf A).

(14) Suppose that
(i) $A \subseteq Z$,
(ii) $f = -\frac{\frac{\text{the function cos}}{\text{the function sin}}}{f_1} - \frac{\frac{1}{\mathrm{id}_Z}}{(\text{the function sin})^{\mathbf{2}}}$,
(iii) $f_1 = \square^2$,
(iv) $Z \subseteq$ dom($\frac{1}{\mathrm{id}_Z}$ (the function cot)),
(v) $Z = \operatorname{dom} f$, and
(vi) f is continuous on A.

Then $\int\limits_A f(x)dx = (\frac{1}{\mathrm{id}_Z}$ (the function cot))(sup A) $-$ ($\frac{1}{\mathrm{id}_Z}$ (the function cot))(inf A).

(15) Suppose that
(i) $A \subseteq Z$,
(ii) $f = \frac{\frac{\text{the function sin}}{\text{the function cos}}}{\mathrm{id}_Z} + \frac{\text{the function ln}}{(\text{the function cos})^{\mathbf{2}}}$,
(iii) $Z \subseteq$ dom((the function ln) (the function tan)),
(iv) $Z = \operatorname{dom} f$, and
(v) f is continuous on A.

Then $\int\limits_A f(x)dx =$ ((the function ln) (the function tan))(sup A) $-$ ((the function ln) (the function tan))(inf A).

(16) Suppose that
(i) $A \subseteq Z$,
(ii) $f = \frac{\frac{\text{the function cos}}{\text{the function sin}}}{\mathrm{id}_Z} - \frac{\text{the function ln}}{(\text{the function sin})^{\mathbf{2}}}$,

(iii) $Z \subseteq \mathrm{dom}(($the function ln$)$ $($the function cot$))$,
(iv) $Z = \mathrm{dom}\, f$, and
(v) f is continuous on A.

Then $\int\limits_A f(x)dx = (($the function ln$)$ $($the function cot$))(\sup A) - (($the function ln$)$ $($the function cot$))(\inf A)$.

(17) Suppose that
(i) $A \subseteq Z$,
(ii) $f = \frac{\text{the function tan}}{\mathrm{id}_Z} + \frac{\text{the function ln}}{(\text{the function cos})^{\mathbf{2}}}$,
(iii) $Z \subseteq \mathrm{dom}(($the function ln$)$ $($the function tan$))$,
(iv) $Z \subseteq \mathrm{dom}\,($the function tan$)$,
(v) $Z = \mathrm{dom}\, f$, and
(vi) f is continuous on A.

Then $\int\limits_A f(x)dx = (($the function ln$)$ $($the function tan$))(\sup A) - (($the function ln$)$ $($the function tan$))(\inf A)$.

(18) Suppose that
(i) $A \subseteq Z$,
(ii) $f = \frac{\text{the function cot}}{\mathrm{id}_Z} - \frac{\text{the function ln}}{(\text{the function sin})^{\mathbf{2}}}$,
(iii) $Z \subseteq \mathrm{dom}(($the function ln$)$ $($the function cot$))$,
(iv) $Z \subseteq \mathrm{dom}\,($the function cot$)$,
(v) $Z = \mathrm{dom}\, f$, and
(vi) f is continuous on A.

Then $\int\limits_A f(x)dx = (($the function ln$)$ $($the function cot$))(\sup A) - (($the function ln$)$ $($the function cot$))(\inf A)$.

(19) Suppose that
(i) $A \subseteq Z$,
(ii) for every x such that $x \in Z$ holds $f_1(x) = 1$,
(iii) $f = \frac{\text{the function arctan}}{\mathrm{id}_Z} + \frac{\text{the function ln}}{f_1 + \square^2}$,
(iv) $Z \subseteq]-1, 1[$,
(v) $Z = \mathrm{dom}\, f$, and
(vi) f is continuous on A.

Then $\int\limits_A f(x)dx = (($the function ln$)$ $($the function arctan$))(\sup A) - (($the function ln$)$ $($the function arctan$))(\inf A)$.

(20) Suppose that
(i) $A \subseteq Z$,
(ii) for every x such that $x \in Z$ holds $f_1(x) = 1$,
(iii) $f = \frac{\text{the function arccot}}{\mathrm{id}_Z} - \frac{\text{the function ln}}{f_1 + \square^2}$,

(iv) $Z \subseteq]-1, 1[$,
(v) $Z = \text{dom} f$, and
(vi) f is continuous on A.

Then $\int\limits_A f(x)dx = $ ((the function ln) (the function arccot))(sup A) $-$ ((the function ln) (the function arccot))(inf A).

(21) Suppose $A \subseteq Z$ and $f = \frac{\text{(the function exp)}\cdot\text{(the function tan)}}{\text{(the function cos)}^2}$ and $Z = \text{dom} f$ and f is continuous on A. Then $\int\limits_A f(x)dx = $ ((the function exp) $\cdot$(the function tan))(sup A) $-$ ((the function exp) $\cdot$(the function tan))(inf A).

(22) Suppose $A \subseteq Z$ and $f = -\frac{\text{(the function exp)}\cdot\text{(the function cot)}}{\text{(the function sin)}^2}$ and $Z = \text{dom} f$ and f is continuous on A. Then $\int\limits_A f(x)dx = $ ((the function exp) $\cdot$(the function cot))(sup A) $-$ ((the function exp) $\cdot$(the function cot))(inf A).

(23) Suppose $Z \subseteq$ dom((the function exp) $\cdot$(the function cot)). Then
(i) $-$(the function exp) $\cdot$ (the function cot) is differentiable on Z, and
(ii) for every x such that $x \in Z$ holds ($-$(the function exp) $\cdot$ (the function cot))$'_{\restriction Z}(x) = \frac{\text{(the function exp)((the function cot)}(x))}{\text{(the function sin)}(x)^2}$.

(24) Suppose $A \subseteq Z$ and $f = \frac{\text{(the function exp)}\cdot\text{(the function cot)}}{\text{(the function sin)}^2}$ and $Z = \text{dom} f$ and f is continuous on A. Then $\int\limits_A f(x)dx = $ ($-$(the function exp) $\cdot$ (the function cot))(sup A) $-$ ($-$(the function exp) $\cdot$ (the function cot))(inf A).

(25) Suppose that
(i) $A \subseteq Z$,
(ii) $f = \frac{1}{\text{id}_Z \text{ ((the function cos)}\cdot\text{(the function ln))}^2}$,
(iii) $Z \subseteq$ dom((the function tan) $\cdot$(the function ln)),
(iv) $Z = \text{dom} f$, and
(v) f is continuous on A.

Then $\int\limits_A f(x)dx = $ ((the function tan) $\cdot$(the function ln))(sup A) $-$ ((the function tan) $\cdot$(the function ln))(inf A).

(26) Suppose that
(i) $A \subseteq Z$,
(ii) $f = -\frac{1}{\text{id}_Z \text{ ((the function sin)}\cdot\text{(the function ln))}^2}$,
(iii) $Z \subseteq$ dom((the function cot) $\cdot$(the function ln)),
(iv) $Z = \text{dom} f$, and
(v) f is continuous on A.

Then $\int\limits_{A} f(x)dx =$ ((the function cot) $\cdot$(the function ln))(sup A) $-$ ((the function cot) $\cdot$(the function ln))(inf A).

(27) Suppose $Z \subseteq$ dom((the function cot) $\cdot$(the function ln)). Then

(i) $-$(the function cot) $\cdot$ (the function ln) is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds ($-$(the function cot) $\cdot$ (the function ln))$'_{\restriction Z}(x) = \frac{1}{x \cdot (\text{the function sin})((\text{the function ln})(x))^{\mathbf{2}}}$.

(28) Suppose that

(i) $A \subseteq Z$,

(ii) $f = \frac{1}{\mathrm{id}_Z\ ((\text{the function sin}) \cdot (\text{the function ln}))^{\mathbf{2}}}$,

(iii) $Z \subseteq$ dom((the function cot) $\cdot$(the function ln)),

(iv) $Z = \operatorname{dom} f$, and

(v) f is continuous on A.

Then $\int\limits_{A} f(x)dx =$ ($-$(the function cot) $\cdot$ (the function ln))(sup A) $-$ ($-$(the function cot) $\cdot$ (the function ln))(inf A).

(29) Suppose that

(i) $A \subseteq Z$,

(ii) $f = \frac{\text{the function exp}}{((\text{the function cos}) \cdot (\text{the function exp}))^{\mathbf{2}}}$,

(iii) $Z \subseteq$ dom((the function tan) $\cdot$(the function exp)),

(iv) $Z = \operatorname{dom} f$, and

(v) f is continuous on A.

Then $\int\limits_{A} f(x)dx =$ ((the function tan) $\cdot$(the function exp))(sup A) $-$ ((the function tan) $\cdot$(the function exp))(inf A).

(30) Suppose that

(i) $A \subseteq Z$,

(ii) $f = -\frac{\text{the function exp}}{((\text{the function sin}) \cdot (\text{the function exp}))^{\mathbf{2}}}$,

(iii) $Z \subseteq$ dom((the function cot) $\cdot$(the function exp)),

(iv) $Z = \operatorname{dom} f$, and

(v) f is continuous on A.

Then $\int\limits_{A} f(x)dx =$ ((the function cot) $\cdot$(the function exp))(sup A) $-$ ((the function cot) $\cdot$(the function exp))(inf A).

(31) Suppose $Z \subseteq$ dom((the function cot) $\cdot$(the function exp)). Then

(i) $-$(the function cot) $\cdot$ (the function exp) is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds ($-$(the function cot) $\cdot$ (the function exp))$'_{\restriction Z}(x) = \frac{(\text{the function exp})(x)}{(\text{the function sin})((\text{the function exp})(x))^{\mathbf{2}}}$.

(32) Suppose that

(i) $A \subseteq Z$,

(ii) $f = \frac{\text{the function exp}}{((\text{the function sin})\cdot(\text{the function exp}))^{\mathbf{2}}}$,
(iii) $Z \subseteq \operatorname{dom}((\text{the function cot}) \cdot (\text{the function exp}))$,
(iv) $Z = \operatorname{dom} f$, and
(v) f is continuous on A.

Then $\int\limits_A f(x)dx = (-(\text{the function cot}) \cdot (\text{the function exp}))(\sup A) - (-(\text{the function cot}) \cdot (\text{the function exp}))(\inf A)$.

(33) Suppose that
(i) $A \subseteq Z$,
(ii) for every x such that $x \in Z$ holds $f(x) = -\frac{1}{x^{\mathbf{2}}\cdot(\text{the function cos})(\frac{1}{x})^{\mathbf{2}}}$,
(iii) $Z \subseteq \operatorname{dom}((\text{the function tan}) \cdot \frac{1}{\mathrm{id}_Z})$,
(iv) $Z = \operatorname{dom} f$, and
(v) f is continuous on A.

Then $\int\limits_A f(x)dx = ((\text{the function tan}) \cdot \frac{1}{\mathrm{id}_Z})(\sup A) - ((\text{the function tan}) \cdot \frac{1}{\mathrm{id}_Z})(\inf A)$.

(34) Suppose $Z \subseteq \operatorname{dom}((\text{the function tan}) \cdot \frac{1}{\mathrm{id}_Z})$. Then
(i) $-(\text{the function tan}) \cdot \frac{1}{\mathrm{id}_Z}$ is differentiable on Z, and
(ii) for every x such that $x \in Z$ holds $(-(\text{the function tan}) \cdot \frac{1}{\mathrm{id}_Z})'_{\restriction Z}(x) = \frac{1}{x^{\mathbf{2}}\cdot(\text{the function cos})(\frac{1}{x})^{\mathbf{2}}}$.

(35) Suppose that
(i) $A \subseteq Z$,
(ii) for every x such that $x \in Z$ holds $f(x) = \frac{1}{x^{\mathbf{2}}\cdot(\text{the function cos})(\frac{1}{x})^{\mathbf{2}}}$,
(iii) $Z \subseteq \operatorname{dom}((\text{the function tan}) \cdot \frac{1}{\mathrm{id}_Z})$,
(iv) $Z = \operatorname{dom} f$, and
(v) f is continuous on A.

Then $\int\limits_A f(x)dx = (-(\text{the function tan}) \cdot \frac{1}{\mathrm{id}_Z})(\sup A) - (-(\text{the function tan}) \cdot \frac{1}{\mathrm{id}_Z})(\inf A)$.

(36) Suppose that
(i) $A \subseteq Z$,
(ii) for every x such that $x \in Z$ holds $f(x) = \frac{1}{x^{\mathbf{2}}\cdot(\text{the function sin})(\frac{1}{x})^{\mathbf{2}}}$,
(iii) $Z \subseteq \operatorname{dom}((\text{the function cot}) \cdot \frac{1}{\mathrm{id}_Z})$,
(iv) $Z = \operatorname{dom} f$, and
(v) f is continuous on A.

Then $\int\limits_A f(x)dx = ((\text{the function cot}) \cdot \frac{1}{\mathrm{id}_Z})(\sup A) - ((\text{the function cot}) \cdot \frac{1}{\mathrm{id}_Z})(\inf A)$.

(37) Suppose that $A \subseteq Z$ and for every x such that $x \in Z$ holds $f_1(x) = 1$ and (the function arctan)$(x) > 0$ and $f = \frac{1}{(f_1+\square^2)\ \text{the function arctan}}$ and $Z \subseteq]-1,1[$ and $Z \subseteq \text{dom}((\text{the function ln}) \cdot (\text{the function arctan}))$ and $Z = \text{dom} f$ and f is continuous on A. Then $\int\limits_A f(x)dx = ((\text{the function ln}) \cdot (\text{the function arctan}))(\sup A) - ((\text{the function ln}) \cdot (\text{the function arctan}))(\inf A)$.

(38) Suppose that $A \subseteq Z$ and $f = n\,\frac{(\square^{n-1})\cdot\text{the function arctan}}{f_1+\square^2}$ and for every x such that $x \in Z$ holds $f_1(x) = 1$ and $Z \subseteq]-1,1[$ and $Z \subseteq \text{dom}((\square^n)\cdot\text{the function arctan})$ and $Z = \text{dom} f$ and f is continuous on A. Then $\int\limits_A f(x)dx = ((\square^n)\cdot\text{the function arctan})(\sup A) - ((\square^n)\cdot\text{the function arctan})(\inf A)$.

(39) Suppose that $A \subseteq Z$ and for every x such that $x \in Z$ holds $f_1(x) = 1$ and $f = -n\,\frac{(\square^{n-1})\cdot\text{the function arccot}}{f_1+\square^2}$ and $Z \subseteq]-1,1[$ and $Z \subseteq \text{dom}((\square^n)\cdot\text{the function arccot})$ and $Z = \text{dom} f$ and f is continuous on A. Then $\int\limits_A f(x)dx = ((\square^n)\cdot\text{the function arccot})(\sup A) - ((\square^n)\cdot\text{the function arccot})(\inf A)$.

(40) Suppose $Z \subseteq \text{dom}((\square^n)\cdot\text{the function arccot})$ and $Z \subseteq]-1,1[$. Then

(i) $-(\square^n)\cdot$ the function arccot is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $(-(\square^n)\cdot\text{the function arccot})'_{\restriction Z}(x) = \frac{n\cdot(\text{the function arccot})(x)^{n-1}}{1+x^{\mathbf{2}}}$.

(41) Suppose that $A \subseteq Z$ and for every x such that $x \in Z$ holds $f_1(x) = 1$ and $f = n\,\frac{(\square^{n-1})\cdot\text{the function arccot}}{f_1+\square^2}$ and $Z \subseteq]-1,1[$ and $Z \subseteq \text{dom}((\square^n)\cdot\text{the function arccot})$ and $Z = \text{dom} f$ and f is continuous on A. Then $\int\limits_A f(x)dx = (-(\square^n)\cdot\text{the function arccot})(\sup A) - (-(\square^n)\cdot\text{the function arccot})(\inf A)$.

(42) Suppose that $A \subseteq Z$ and for every x such that $x \in Z$ holds $f_1(x) = 1$ and $f = \frac{\text{the function arctan}}{f_1+\square^2}$ and $Z \subseteq]-1,1[$ and $Z \subseteq \text{dom}((\square^2)\cdot\text{the function arctan})$ and $Z = \text{dom} f$ and f is continuous on A. Then $\int\limits_A f(x)dx = (\frac{1}{2}\,((\square^2)\cdot\text{the function arctan}))(\sup A) - (\frac{1}{2}\,((\square^2)\cdot\text{the function arctan}))(\inf A)$.

(43) Suppose that $A \subseteq Z$ and for every x such that $x \in Z$ holds $f_1(x) = 1$ and $f = -\frac{\text{the function arccot}}{f_1+\square^2}$ and $Z \subseteq]-1,1[$ and $Z \subseteq \text{dom}((\square^2)\cdot\text{the function arccot})$ and $Z = \text{dom} f$ and f is continuous on A. Then $\int\limits_A f(x)dx = (\frac{1}{2}\,((\square^2)\cdot\text{the function arccot}))(\sup A) - (\frac{1}{2}\,((\square^2)\cdot\text{the function arccot}))(\inf A)$.

(44) Suppose $Z \subseteq \text{dom}((\square^2)\cdot\text{the function arccot})$ and $Z \subseteq]-1,1[$. Then

(i) $-\frac{1}{2}\,((\square^2)\cdot\text{the function arccot})$ is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds

$(-\frac{1}{2}\,((\Box^2)\cdot \text{the function arccot}))'_{\restriction Z}(x) = \frac{(\text{the function arccot})(x)}{1+x^{\mathbf{2}}}$.

(45) Suppose that $A \subseteq Z$ and for every x such that $x \in Z$ holds $f_1(x) = 1$ and $f = \frac{\text{the function arccot}}{f_1+\Box^2}$ and $Z \subseteq]-1,1[$ and $Z \subseteq \operatorname{dom}((\Box^2)\cdot \text{the function arccot})$ and $Z = \operatorname{dom} f$ and f is continuous on A. Then $\int\limits_A f(x)dx = (-\frac{1}{2}\,((\Box^2)\cdot \text{the function arccot}))(\sup A) - (-\frac{1}{2}\,((\Box^2)\cdot \text{the function arccot}))(\inf A)$.

(46) Suppose that
- (i) $A \subseteq Z$,
- (ii) for every x such that $x \in Z$ holds $f_1(x) = 1$,
- (iii) $f = (\text{the function arctan}) + \frac{\mathrm{id}_Z}{f_1+\Box^2}$,
- (iv) $Z \subseteq]-1,1[$,
- (v) $Z = \operatorname{dom} f$, and
- (vi) f is continuous on A.

Then $\int\limits_A f(x)dx = (\mathrm{id}_Z\,\text{the function arctan})(\sup A) - (\mathrm{id}_Z\,\text{the function arctan})(\inf A)$.

(47) Suppose that
- (i) $A \subseteq Z$,
- (ii) for every x such that $x \in Z$ holds $f_1(x) = 1$,
- (iii) $f = (\text{the function arccot}) - \frac{\mathrm{id}_Z}{f_1+\Box^2}$,
- (iv) $Z \subseteq]-1,1[$,
- (v) $Z = \operatorname{dom} f$, and
- (vi) f is continuous on A.

Then $\int\limits_A f(x)dx = (\mathrm{id}_Z\,\text{the function arccot})(\sup A) - (\mathrm{id}_Z\,\text{the function arccot})(\inf A)$.

(48) Suppose that
- (i) $A \subseteq Z$,
- (ii) $Z \subseteq]-1,1[$,
- (iii) $f = \frac{(\text{the function exp})\cdot(\text{the function arctan})}{f_1+\Box^2}$,
- (iv) for every x such that $x \in Z$ holds $f_1(x) = 1$,
- (v) $Z = \operatorname{dom} f$, and
- (vi) f is continuous on A.

Then $\int\limits_A f(x)dx = ((\text{the function exp})\cdot(\text{the function arctan}))(\sup A) - ((\text{the function exp})\cdot(\text{the function arctan}))(\inf A)$.

(49) Suppose that
- (i) $A \subseteq Z$,
- (ii) $Z \subseteq]-1,1[$,

(iii) $f = -\frac{(\text{the function exp})\cdot(\text{the function arccot})}{f_1+\square^2}$,
(iv) for every x such that $x \in Z$ holds $f_1(x) = 1$,
(v) $Z = \mathrm{dom}\, f$, and
(vi) f is continuous on A.

Then $\int_A f(x)dx$ = ((the function exp) ·(the function arccot))(sup A) − ((the function exp) ·(the function arccot))(inf A).

(50) Suppose $Z \subseteq$ dom((the function exp) ·(the function arccot)) and $Z \subseteq]-1,1[$. Then
(i) −(the function exp) · (the function arccot) is differentiable on Z, and
(ii) for every x such that $x \in Z$ holds (−(the function exp) · (the function arccot))$'_{\restriction Z}(x) = \frac{(\text{the function exp})((\text{the function arccot})(x))}{1+x^2}$.

(51) Suppose that
(i) $A \subseteq Z$,
(ii) $Z \subseteq]-1,1[$,
(iii) $f = \frac{(\text{the function exp})\cdot(\text{the function arccot})}{f_1+\square^2}$,
(iv) for every x such that $x \in Z$ holds $f_1(x) = 1$,
(v) $Z = \mathrm{dom}\, f$, and
(vi) f is continuous on A.

Then $\int_A f(x)dx$ = (−(the function exp) · (the function arccot))(sup A) − (−(the function exp) · (the function arccot))(inf A).

(52) Suppose that $A \subseteq Z \subseteq$ dom((the function ln) ·$(f_1 + f_2)$) and $f = \frac{\mathrm{id}_Z}{f_1+f_2}$ and $f_2 = \square^2$ and for every x such that $x \in Z$ holds $f_1(x) = 1$ and $Z = \mathrm{dom}\, f$ and f is continuous on A. Then $\int_A f(x)dx$ = ($\frac{1}{2}$ ((the function ln) ·$(f_1 + f_2)$))(sup A)− ($\frac{1}{2}$ ((the function ln) ·$(f_1 + f_2)$))(inf A).

(53) Suppose that $A \subseteq Z \subseteq$ dom((the function ln) ·(f_1+f_2)) and $f = \frac{\mathrm{id}_Z}{a\,(f_1+f_2)}$ and for every x such that $x \in Z$ holds $h(x) = \frac{x}{a}$ and $f_1(x) = 1$ and $a \neq 0$ and $f_2 = (\square^2) \cdot h$ and $Z = \mathrm{dom}\, f$ and f is continuous on A. Then $\int_A f(x)dx$ = ($\frac{a}{2}$ ((the function ln) ·$(f_1 + f_2)$))(sup A) − ($\frac{a}{2}$ ((the function ln) ·$(f_1 + f_2)$))(inf A).

(54) Suppose $Z \subseteq$ dom($\frac{1}{\mathrm{id}_Z}$ the function arctan) and $Z \subseteq]-1,1[$. Then
(i) $-\frac{1}{\mathrm{id}_Z}$ the function arctan is differentiable on Z, and
(ii) for every x such that $x \in Z$ holds ($-\frac{1}{\mathrm{id}_Z}$ the function arctan)$'_{\restriction Z}(x) = \frac{(\text{the function arctan})(x)}{x^2} - \frac{1}{x\cdot(1+x^2)}$.

(55) Suppose $Z \subseteq$ dom($\frac{1}{\mathrm{id}_Z}$ the function arccot) and $Z \subseteq]-1,1[$. Then
(i) $-\frac{1}{\mathrm{id}_Z}$ the function arccot is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $(-\frac{1}{\mathrm{id}_Z}$ the function arccot$)'_{\restriction Z}(x) = \frac{(\text{the function arccot})(x)}{x^2} + \frac{1}{x \cdot (1+x^2)}$.

(56) Suppose that $A \subseteq Z$ and for every x such that $x \in Z$ holds $f_1(x) = 1$ and $f = \frac{\text{the function arctan}}{\square^2} - \frac{1}{\mathrm{id}_Z\ (f_1+\square^2)}$ and $Z \subseteq \mathrm{dom}(\frac{1}{\mathrm{id}_Z}$ the function arctan) and $Z \subseteq]-1,1[$ and $Z = \mathrm{dom}\, f$ and f is continuous on A. Then $\int\limits_A f(x)dx = (-\frac{1}{\mathrm{id}_Z}$ the function arctan$)(\sup A) - (-\frac{1}{\mathrm{id}_Z}$ the function arctan$)(\inf A)$.

(57) Suppose that $A \subseteq Z$ and for every x such that $x \in Z$ holds $f_1(x) = 1$ and $f = \frac{\text{the function arccot}}{\square^2} + \frac{1}{\mathrm{id}_Z\ (f_1+\square^2)}$ and $Z \subseteq \mathrm{dom}(\frac{1}{\mathrm{id}_Z}$ the function arccot) and $Z \subseteq]-1,1[$ and $Z = \mathrm{dom}\, f$ and f is continuous on A. Then $\int\limits_A f(x)dx = (-\frac{1}{\mathrm{id}_Z}$ the function arccot$)(\sup A) - (-\frac{1}{\mathrm{id}_Z}$ the function arccot$)(\inf A)$.

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