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On the Partial Product and Partial Sum of Series and Related Basic Inequalities

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Summary. This article introduced some important inequalities on partial sum and partial product, as well as some basic inequalities.

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The notation and terminology used in this paper are introduced in the following papers: [2], [1], [9], [6], [3], [5], [7], [8], and [4].

For simplicity, we adopt the following rules: a, b, c, d are positive real numbers, m, u, w, x, y, z are real numbers, n, k are natural numbers, and s, s_1 are sequences of real numbers.

Next we state a number of propositions:

(1) $(a+b)\cdot(\frac{1}{a}+\frac{1}{b}) \geq 4$.

(2) $a^4+b^4 \geq a^3\cdot b+a\cdot b^3$.

(3) If $a<b$, then $1<\frac{b+c}{a+c}$.

(4) If $a<b$, then $\frac{a}{b}<\sqrt{\frac{a}{b}}$.

(5) If $a<b$, then $\sqrt{\frac{a}{b}}<\frac{b+\sqrt{\frac{a^{\mathbf{2}}+b^{\mathbf{2}}}{2}}}{a+\sqrt{\frac{a^{\mathbf{2}}+b^{\mathbf{2}}}{2}}}$.

(6) If $a<b$, then $\frac{a}{b}<\frac{b+\sqrt{\frac{a^{\mathbf{2}}+b^{\mathbf{2}}}{2}}}{a+\sqrt{\frac{a^{\mathbf{2}}+b^{\mathbf{2}}}{2}}}$.

(7) $\frac{2}{\frac{1}{a}+\frac{1}{b}} \leq \sqrt{a\cdot b}$.

(8) $\frac{a+b}{2} \leq \sqrt{\frac{a^{\mathbf{2}}+b^{\mathbf{2}}}{2}}$.

(9) $x+y \leq \sqrt{2\cdot(x^{\mathbf{2}}+y^{\mathbf{2}})}$.

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(10) $\frac{2}{\frac{1}{a}+\frac{1}{b}} \leq \frac{a+b}{2}$.

(11) $\sqrt{a \cdot b} \leq \sqrt{\frac{a^{2}+b^{2}}{2}}$.

(12) $\frac{2}{\frac{1}{a}+\frac{1}{b}} \leq \sqrt{\frac{a^{2}+b^{2}}{2}}$.

(13) If $|x| < 1$ and $|y| < 1$, then $|\frac{x+y}{1+x \cdot y}| \leq 1$.

(14) $\frac{|x+y|}{1+|x+y|} \leq \frac{|x|}{1+|x|} + \frac{|y|}{1+|y|}$.

(15) $\frac{a}{a+b+d} + \frac{b}{a+b+c} + \frac{c}{b+c+d} + \frac{d}{a+c+d} > 1$.

(16) $\frac{a}{a+b+d} + \frac{b}{a+b+c} + \frac{c}{b+c+d} + \frac{d}{a+c+d} < 2$.

(17) If $a+b > c$ and $b+c > a$ and $a+c > b$, then $\frac{1}{(a+b)-c} + \frac{1}{(b+c)-a} + \frac{1}{(c+a)-b} \geq \frac{9}{a+b+c}$.

(18) $\sqrt{(a+b) \cdot (c+d)} \geq \sqrt{a \cdot c} + \sqrt{b \cdot d}$.

(19) $(a \cdot b + c \cdot d) \cdot (a \cdot c + b \cdot d) \geq 4 \cdot a \cdot b \cdot c \cdot d$.

(20) $\frac{a}{b} + \frac{b}{c} + \frac{c}{a} \geq 3$.

(21) If $a \cdot b + b \cdot c + c \cdot a = 1$, then $a + b + c \geq \sqrt{3}$.

(22) $\frac{(b+c)-a}{a} + \frac{(c+a)-b}{b} + \frac{(a+b)-c}{c} \geq 3$.

(23) $(a + \frac{1}{a}) \cdot (b + \frac{1}{b}) \geq (\sqrt{a \cdot b} + \frac{1}{\sqrt{a \cdot b}})^{2}$.

(24) $\frac{b \cdot c}{a} + \frac{a \cdot c}{b} + \frac{a \cdot b}{c} \geq a + b + c$.

(25) If $x > y$ and $y > z$, then $x^{2} \cdot y + y^{2} \cdot z + z^{2} \cdot x > x \cdot y^{2} + y \cdot z^{2} + z \cdot x^{2}$.

(26) If $a > b$ and $b > c$, then $\frac{b}{a-b} > \frac{c}{a-c}$.

(27) If $b > a$ and $c > d$, then $\frac{c}{c+a} > \frac{d}{d+b}$.

(28) $m \cdot x + z \cdot y \leq \sqrt{m^{2} + z^{2}} \cdot \sqrt{x^{2} + y^{2}}$.

(29) $(m \cdot x + u \cdot y + w \cdot z)^{2} \leq (m^{2} + u^{2} + w^{2}) \cdot (x^{2} + y^{2} + z^{2})$.

(30) $\frac{9 \cdot a \cdot b \cdot c}{a^{2}+b^{2}+c^{2}} \leq a + b + c$.

(31) $a + b + c \leq \sqrt{\frac{a^{2}+a \cdot b+b^{2}}{3}} + \sqrt{\frac{b^{2}+b \cdot c+c^{2}}{3}} + \sqrt{\frac{c^{2}+c \cdot a+a^{2}}{3}}$.

(32) $\sqrt{\frac{a^{2}+a \cdot b+b^{2}}{3}} + \sqrt{\frac{b^{2}+b \cdot c+c^{2}}{3}} + \sqrt{\frac{c^{2}+c \cdot a+a^{2}}{3}} \leq \sqrt{\frac{a^{2}+b^{2}}{2}} + \sqrt{\frac{b^{2}+c^{2}}{2}} + \sqrt{\frac{c^{2}+a^{2}}{2}}$.

(33) $\sqrt{\frac{a^{2}+b^{2}}{2}} + \sqrt{\frac{b^{2}+c^{2}}{2}} + \sqrt{\frac{c^{2}+a^{2}}{2}} \leq \sqrt{3 \cdot (a^{2} + b^{2} + c^{2})}$.

(34) $\sqrt{3 \cdot (a^{2} + b^{2} + c^{2})} \leq \frac{b \cdot c}{a} + \frac{c \cdot a}{b} + \frac{a \cdot b}{c}$.

(35) If $a + b = 1$, then $(\frac{1}{a^{2}} - 1) \cdot (\frac{1}{b^{2}} - 1) \geq 9$.

(36) If $a + b = 1$, then $a \cdot b + \frac{1}{a \cdot b} \geq \frac{17}{4}$.

(37) If $a + b + c = 1$, then $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} \geq 9$.

(38) If $a + b + c = 1$, then $(\frac{1}{a} - 1) \cdot (\frac{1}{b} - 1) \cdot (\frac{1}{c} - 1) \geq 8$.

(39) If $a + b + c = 1$, then $(1 + \frac{1}{a}) \cdot (1 + \frac{1}{b}) \cdot (1 + \frac{1}{c}) \geq 64$.

(40) If $x + y + z = 1$, then $x^{2} + y^{2} + z^{2} \geq \frac{1}{3}$.

(41) If $x + y + z = 1$, then $x \cdot y + y \cdot z + z \cdot x \leq \frac{1}{3}$.

(42) If $a \cdot b \cdot c = 1$, then $\sqrt{a} + \sqrt{b} + \sqrt{c} \leq \frac{1}{a} + \frac{1}{b} + \frac{1}{c}$.

(43) If $a > b$ and $b > c$, then $a^{2 \cdot a} \cdot b^{2 \cdot b} \cdot c^{2 \cdot c} > a^{b+c} \cdot b^{a+c} \cdot c^{a+b}$.

(44) If $n \geq 1$, then $a^{n+1} + b^{n+1} \geq a^n \cdot b + a \cdot b^n$.

(45) If $a^{\mathbf{2}} + b^{\mathbf{2}} = c^{\mathbf{2}}$ and $n \geq 3$, then $a^{n+2} + b^{n+2} < c^{n+2}$.

(46) If $n \geq 1$, then $(1 + \frac{1}{n+1})^n < (1 + \frac{1}{n})^{n+1}$.

(47) If $n \geq 1$ and $k \geq 1$, then $(a^k + b^k) \cdot (a^n + b^n) \leq 2 \cdot (a^{k+n} + b^{k+n})$.

(48) If for every n holds $s(n) = \frac{1}{\sqrt{n+1}}$, then for every n holds $(\sum_{\alpha=0}^{\kappa} s(\alpha))_{\kappa \in \mathbb{N}}(n) < 2 \cdot \sqrt{n+1}$.

(49) If for every n holds $s(n) = \frac{1}{(n+1)^{\mathbf{2}}}$, then for every n holds $(\sum_{\alpha=0}^{\kappa} s(\alpha))_{\kappa \in \mathbb{N}}(n) \leq 2 - \frac{1}{n+1}$.

(50) If for every n holds $s(n) = \frac{1}{(n+1)^{\mathbf{2}}}$, then $(\sum_{\alpha=0}^{\kappa} s(\alpha))_{\kappa \in \mathbb{N}}(n) < 2$.

(51) If for every n holds $s(n) < 1$, then for every n holds $(\sum_{\alpha=0}^{\kappa} s(\alpha))_{\kappa \in \mathbb{N}}(n) < n + 1$.

(52) If for every n holds $s(n) > 0$ and $s(n) < 1$, then for every n holds (the partial product of $s)(n) \geq (\sum_{\alpha=0}^{\kappa} s(\alpha))_{\kappa \in \mathbb{N}}(n) - n$.

(53) If for every n holds $s(n) > 0$ and $s_1(n) = \frac{1}{s(n)}$, then for every n holds $(\sum_{\alpha=0}^{\kappa} (s_1)(\alpha))_{\kappa \in \mathbb{N}}(n) > 0$.

(54) If for every n holds $s(n) > 0$ and $s_1(n) = \frac{1}{s(n)}$, then for every n holds $(\sum_{\alpha=0}^{\kappa} s(\alpha))_{\kappa \in \mathbb{N}}(n) \cdot (\sum_{\alpha=0}^{\kappa} (s_1)(\alpha))_{\kappa \in \mathbb{N}}(n) \geq (n+1)^{\mathbf{2}}$.

(55) If for every n such that $n \geq 1$ holds $s(n) = \sqrt{n}$ and $s(0) = 0$, then for every n such that $n \geq 1$ holds $(\sum_{\alpha=0}^{\kappa} s(\alpha))_{\kappa \in \mathbb{N}}(n) < \frac{1}{6} \cdot (4 \cdot n + 3) \cdot \sqrt{n}$.

(56) If for every n such that $n \geq 1$ holds $s(n) = \sqrt{n}$ and $s(0) = 0$, then for every n such that $n \geq 1$ holds $(\sum_{\alpha=0}^{\kappa} s(\alpha))_{\kappa \in \mathbb{N}}(n) > \frac{2}{3} \cdot n \cdot \sqrt{n}$.

(57) Suppose that for every n such that $n \geq 1$ holds $s(n) = 1 + \frac{1}{2 \cdot n+1}$ and $s(0) = 1$. Let given n. If $n \geq 1$, then (the partial product of $s)(n) > \frac{1}{2} \cdot \sqrt{2 \cdot n + 3}$.

(58) If for every n such that $n \geq 1$ holds $s(n) = \sqrt{n \cdot (n+1)}$ and $s(0) = 0$, then for every n such that $n \geq 1$ holds $(\sum_{\alpha=0}^{\kappa} s(\alpha))_{\kappa \in \mathbb{N}}(n) > \frac{n \cdot (n+1)}{2}$.

References

[1] Grzegorz Bancerek. The fundamental properties of natural numbers. *Formalized Mathematics*, 1(**1**):41–46, 1990.

[2] Grzegorz Bancerek. The ordinal numbers. *Formalized Mathematics*, 1(**1**):91–96, 1990.

[3] Czesław Byliński. The complex numbers. *Formalized Mathematics*, 1(**3**):507–513, 1990.

[4] Fuguo Ge and Xiquan Liang. On the partial product of series and related basic inequalities. *Formalized Mathematics*, 13(**3**):413–416, 2005.

[5] Jarosław Kotowicz. Real sequences and basic operations on them. *Formalized Mathematics*, 1(**2**):269–272, 1990.

[6] Rafał Kwiatek. Factorial and Newton coefficients. *Formalized Mathematics*, 1(**5**):887–890, 1990.

[7] Konrad Raczkowski and Andrzej Nędzusiak. Real exponents and logarithms. *Formalized Mathematics*, 2(**2**):213–216, 1991.
[8] Konrad Raczkowski and Andrzej Nędzusiak. Series. *Formalized Mathematics*, 2(**4**):449–452, 1991.
[9] Andrzej Trybulec and Czesław Byliński. Some properties of real numbers. *Formalized Mathematics*, 1(**3**):445–449, 1990.

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