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Several Differentiable Formulas of Special Functions

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Summary. In this article, we give several differentiable formulas of special functions. There are some specific composite functions consisting of rational functions, irrational functions, trigonometric functions, exponential functions or logarithmic functions.

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The notation and terminology used in this paper have been introduced in the following articles: [13], [15], [16], [1], [4], [10], [12], [3], [6], [9], [7], [8], [11], [17], [5], [14], and [2].

For simplicity, we follow the rules: x, a, b, c denote real numbers, n denotes a natural number, Z denotes an open subset of $\mathbb{R}$, and f, f_1, f_2 denote partial functions from $\mathbb{R}$ to $\mathbb{R}$.

One can prove the following propositions:

(1) Suppose $Z \subseteq \mathrm{dom}(\log\_(e) \cdot f)$ and for every x such that $x \in Z$ holds $f(x) = a + x$ and $f(x) > 0$. Then $\log\_(e) \cdot f$ is differentiable on Z and for every x such that $x \in Z$ holds $(\log\_(e) \cdot f)'_{\restriction Z}(x) = \frac{1}{a+x}$.

(2) Suppose $Z \subseteq \mathrm{dom}(\log\_(e) \cdot f)$ and for every x such that $x \in Z$ holds $f(x) = x - a$ and $f(x) > 0$. Then $\log\_(e) \cdot f$ is differentiable on Z and for every x such that $x \in Z$ holds $(\log\_(e) \cdot f)'_{\restriction Z}(x) = \frac{1}{x-a}$.

(3) Suppose $Z \subseteq \mathrm{dom}(-\log\_(e) \cdot f)$ and for every x such that $x \in Z$ holds $f(x) = a - x$ and $f(x) > 0$. Then $-\log\_(e) \cdot f$ is differentiable on Z and for every x such that $x \in Z$ holds $(-\log\_(e) \cdot f)'_{\restriction Z}(x) = \frac{1}{a-x}$.

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(4) Suppose $Z \subseteq \mathrm{dom}(\mathrm{id}_Z - a\,f)$ and $f = \log\_(e) \cdot f_1$ and for every x such that $x \in Z$ holds $f_1(x) = a + x$ and $f_1(x) > 0$. Then $\mathrm{id}_Z - a\,f$ is differentiable on Z and for every x such that $x \in Z$ holds $(\mathrm{id}_Z - a\,f)'_{\restriction Z}(x) = \frac{x}{a+x}$.

(5) Suppose $Z \subseteq \mathrm{dom}((2 \cdot a)\,f - \mathrm{id}_Z)$ and $f = \log\_(e) \cdot f_1$ and for every x such that $x \in Z$ holds $f_1(x) = a + x$ and $f_1(x) > 0$. Then $(2 \cdot a)\,f - \mathrm{id}_Z$ is differentiable on Z and for every x such that $x \in Z$ holds $((2 \cdot a)\,f - \mathrm{id}_Z)'_{\restriction Z}(x) = \frac{a-x}{a+x}$.

(6) Suppose $Z \subseteq \mathrm{dom}(\mathrm{id}_Z - (2 \cdot a)\,f)$ and $f = \log\_(e) \cdot f_1$ and for every x such that $x \in Z$ holds $f_1(x) = x + a$ and $f_1(x) > 0$. Then $\mathrm{id}_Z - (2 \cdot a)\,f$ is differentiable on Z and for every x such that $x \in Z$ holds $(\mathrm{id}_Z - (2 \cdot a)\,f)'_{\restriction Z}(x) = \frac{x-a}{x+a}$.

(7) Suppose $Z \subseteq \mathrm{dom}(\mathrm{id}_Z + (2 \cdot a)\,f)$ and $f = \log\_(e) \cdot f_1$ and for every x such that $x \in Z$ holds $f_1(x) = x - a$ and $f_1(x) > 0$. Then $\mathrm{id}_Z + (2 \cdot a)\,f$ is differentiable on Z and for every x such that $x \in Z$ holds $(\mathrm{id}_Z + (2 \cdot a)\,f)'_{\restriction Z}(x) = \frac{x+a}{x-a}$.

(8) Suppose $Z \subseteq \mathrm{dom}(\mathrm{id}_Z + (a - b)\,f)$ and $f = \log\_(e) \cdot f_1$ and for every x such that $x \in Z$ holds $f_1(x) = x + b$ and $f_1(x) > 0$. Then $\mathrm{id}_Z + (a - b)\,f$ is differentiable on Z and for every x such that $x \in Z$ holds $(\mathrm{id}_Z + (a - b)\,f)'_{\restriction Z}(x) = \frac{x+a}{x+b}$.

(9) Suppose $Z \subseteq \mathrm{dom}(\mathrm{id}_Z + (a + b)\,f)$ and $f = \log\_(e) \cdot f_1$ and for every x such that $x \in Z$ holds $f_1(x) = x - b$ and $f_1(x) > 0$. Then $\mathrm{id}_Z + (a + b)\,f$ is differentiable on Z and for every x such that $x \in Z$ holds $(\mathrm{id}_Z + (a + b)\,f)'_{\restriction Z}(x) = \frac{x+a}{x-b}$.

(10) Suppose $Z \subseteq \mathrm{dom}(\mathrm{id}_Z - (a + b)\,f)$ and $f = \log\_(e) \cdot f_1$ and for every x such that $x \in Z$ holds $f_1(x) = x + b$ and $f_1(x) > 0$. Then $\mathrm{id}_Z - (a + b)\,f$ is differentiable on Z and for every x such that $x \in Z$ holds $(\mathrm{id}_Z - (a + b)\,f)'_{\restriction Z}(x) = \frac{x-a}{x+b}$.

(11) Suppose $Z \subseteq \mathrm{dom}(\mathrm{id}_Z + (b - a)\,f)$ and $f = \log\_(e) \cdot f_1$ and for every x such that $x \in Z$ holds $f_1(x) = x - b$ and $f_1(x) > 0$. Then $\mathrm{id}_Z + (b - a)\,f$ is differentiable on Z and for every x such that $x \in Z$ holds $(\mathrm{id}_Z + (b - a)\,f)'_{\restriction Z}(x) = \frac{x-a}{x-b}$.

(12) Suppose $Z \subseteq \mathrm{dom}(f_1 + c\,f_2)$ and for every x such that $x \in Z$ holds $f_1(x) = a + b \cdot x$ and $f_2 = \frac{2}{\mathbb{Z}}$. Then $f_1 + c\,f_2$ is differentiable on Z and for every x such that $x \in Z$ holds $(f_1 + c\,f_2)'_{\restriction Z}(x) = b + 2 \cdot c \cdot x$.

(13) Suppose $Z \subseteq \mathrm{dom}(\log\_(e) \cdot (f_1 + c\,f_2))$ and $f_2 = \frac{2}{\mathbb{Z}}$ and for every x such that $x \in Z$ holds $f_1(x) = a + b \cdot x$ and $(f_1 + c\,f_2)(x) > 0$. Then $\log\_(e) \cdot (f_1 + c\,f_2)$ is differentiable on Z and for every x such that $x \in Z$ holds $(\log\_(e) \cdot (f_1 + c\,f_2))'_{\restriction Z}(x) = \frac{b+2 \cdot c \cdot x}{a+b \cdot x+c \cdot x^2}$.

(14) Suppose $Z \subseteq \mathrm{dom}\, f$ and for every x such that $x \in Z$ holds $f(x) = a + x$ and $f(x) \neq 0$. Then $\frac{1}{f}$ is differentiable on Z and for every x such that

$x \in Z$ holds $(\frac{1}{f})'_{\restriction Z}(x) = -\frac{1}{(a+x)^{\mathbf{2}}}$.

(15) Suppose $Z \subseteq \mathrm{dom}((-1)\,\frac{1}{f})$ and for every x such that $x \in Z$ holds $f(x) = a + x$ and $f(x) \neq 0$. Then $(-1)\,\frac{1}{f}$ is differentiable on Z and for every x such that $x \in Z$ holds $((-1)\,\frac{1}{f})'_{\restriction Z}(x) = \frac{1}{(a+x)^{\mathbf{2}}}$.

(16) Suppose $Z \subseteq \mathrm{dom}\, f$ and for every x such that $x \in Z$ holds $f(x) = a - x$ and $f(x) \neq 0$. Then $\frac{1}{f}$ is differentiable on Z and for every x such that $x \in Z$ holds $(\frac{1}{f})'_{\restriction Z}(x) = \frac{1}{(a-x)^{\mathbf{2}}}$.

(17) Suppose $Z \subseteq \mathrm{dom}(f_1 + f_2)$ and for every x such that $x \in Z$ holds $f_1(x) = a^{\mathbf{2}}$ and $f_2 = {}^{2}_{\mathbb{Z}}$. Then $f_1 + f_2$ is differentiable on Z and for every x such that $x \in Z$ holds $(f_1 + f_2)'_{\restriction Z}(x) = 2 \cdot x$.

(18) Suppose $Z \subseteq \mathrm{dom}(\log\_(e) \cdot (f_1 + f_2))$ and $f_2 = {}^{2}_{\mathbb{Z}}$ and for every x such that $x \in Z$ holds $f_1(x) = a^{\mathbf{2}}$ and $(f_1 + f_2)(x) > 0$. Then $\log\_(e) \cdot (f_1 + f_2)$ is differentiable on Z and for every x such that $x \in Z$ holds $(\log\_(e) \cdot (f_1 + f_2))'_{\restriction Z}(x) = \frac{2 \cdot x}{a^{\mathbf{2}} + x^2}$.

(19) Suppose $Z \subseteq \mathrm{dom}(-\log\_(e) \cdot (f_1 - f_2))$ and $f_2 = {}^{2}_{\mathbb{Z}}$ and for every x such that $x \in Z$ holds $f_1(x) = a^{\mathbf{2}}$ and $(f_1 - f_2)(x) > 0$. Then $-\log\_(e) \cdot (f_1 - f_2)$ is differentiable on Z and for every x such that $x \in Z$ holds $(-\log\_(e) \cdot (f_1 - f_2))'_{\restriction Z}(x) = \frac{2 \cdot x}{a^{\mathbf{2}} - x^2}$.

(20) Suppose $Z \subseteq \mathrm{dom}(f_1 + f_2)$ and for every x such that $x \in Z$ holds $f_1(x) = a$ and $f_2 = {}^{3}_{\mathbb{Z}}$. Then $f_1 + f_2$ is differentiable on Z and for every x such that $x \in Z$ holds $(f_1 + f_2)'_{\restriction Z}(x) = 3 \cdot x^2$.

(21) Suppose $Z \subseteq \mathrm{dom}(\log\_(e) \cdot (f_1 + f_2))$ and $f_2 = {}^{3}_{\mathbb{Z}}$ and for every x such that $x \in Z$ holds $f_1(x) = a$ and $(f_1 + f_2)(x) > 0$. Then $\log\_(e) \cdot (f_1 + f_2)$ is differentiable on Z and for every x such that $x \in Z$ holds $(\log\_(e) \cdot (f_1 + f_2))'_{\restriction Z}(x) = \frac{3 \cdot x^2}{a + x^3}$.

(22) Suppose $Z \subseteq \mathrm{dom}(\log\_(e) \cdot \frac{f_1}{f_2})$ and for every x such that $x \in Z$ holds $f_1(x) = a + x$ and $f_1(x) > 0$ and $f_2(x) = a - x$ and $f_2(x) > 0$. Then $\log\_(e) \cdot \frac{f_1}{f_2}$ is differentiable on Z and for every x such that $x \in Z$ holds $(\log\_(e) \cdot \frac{f_1}{f_2})'_{\restriction Z}(x) = \frac{2 \cdot a}{a^{\mathbf{2}} - x^{\mathbf{2}}}$.

(23) Suppose $Z \subseteq \mathrm{dom}(\log\_(e) \cdot \frac{f_1}{f_2})$ and for every x such that $x \in Z$ holds $f_1(x) = x - a$ and $f_1(x) > 0$ and $f_2(x) = x + a$ and $f_2(x) > 0$. Then $\log\_(e) \cdot \frac{f_1}{f_2}$ is differentiable on Z and for every x such that $x \in Z$ holds $(\log\_(e) \cdot \frac{f_1}{f_2})'_{\restriction Z}(x) = \frac{2 \cdot a}{x^{\mathbf{2}} - a^{\mathbf{2}}}$.

(24) Suppose $Z \subseteq \mathrm{dom}(\log\_(e) \cdot \frac{f_1}{f_2})$ and for every x such that $x \in Z$ holds $f_1(x) = x - a$ and $f_1(x) > 0$ and $f_2(x) = x - b$ and $f_2(x) > 0$. Then $\log\_(e) \cdot \frac{f_1}{f_2}$ is differentiable on Z and for every x such that $x \in Z$ holds $(\log\_(e) \cdot \frac{f_1}{f_2})'_{\restriction Z}(x) = \frac{a-b}{(x-a) \cdot (x-b)}$.

(25) Suppose $Z \subseteq \mathrm{dom}(\frac{1}{a-b}\, f)$ and $f = \log\_(e) \cdot \frac{f_1}{f_2}$ and for every x such that

$x \in Z$ holds $f_1(x) = x - a$ and $f_1(x) > 0$ and $f_2(x) = x - b$ and $f_2(x) > 0$ and $a - b \neq 0$. Then $\frac{1}{a-b} f$ is differentiable on Z and for every x such that $x \in Z$ holds $(\frac{1}{a-b} f)'_{\restriction Z}(x) = \frac{1}{(x-a)\cdot(x-b)}$.

(26) Suppose $Z \subseteq \mathrm{dom}(\log\_(e) \cdot \frac{f_1}{f_2})$ and $f_2 = \frac{2}{\mathbb{Z}}$ and for every x such that $x \in Z$ holds $f_1(x) = x - a$ and $f_1(x) > 0$ and $f_2(x) > 0$ and $x \neq 0$. Then $\log\_(e) \cdot \frac{f_1}{f_2}$ is differentiable on Z and for every x such that $x \in Z$ holds $(\log\_(e) \cdot \frac{f_1}{f_2})'_{\restriction Z}(x) = \frac{2\cdot a - x}{x\cdot(x-a)}$.

(27) Suppose $Z \subseteq \mathrm{dom}((\overset{\frac{3}{2}}{\mathbb{R}}) \cdot f)$ and for every x such that $x \in Z$ holds $f(x) = a + x$ and $f(x) > 0$. Then $(\overset{\frac{3}{2}}{\mathbb{R}}) \cdot f$ is differentiable on Z and for every x such that $x \in Z$ holds $((\overset{\frac{3}{2}}{\mathbb{R}}) \cdot f)'_{\restriction Z}(x) = \frac{3}{2} \cdot (a + x)_{\mathbb{R}}^{\frac{1}{2}}$.

(28) Suppose $Z \subseteq \mathrm{dom}(\frac{2}{3}\,((\overset{\frac{3}{2}}{\mathbb{R}}) \cdot f))$ and for every x such that $x \in Z$ holds $f(x) = a + x$ and $f(x) > 0$. Then $\frac{2}{3}\,((\overset{\frac{3}{2}}{\mathbb{R}}) \cdot f)$ is differentiable on Z and for every x such that $x \in Z$ holds $(\frac{2}{3}\,((\overset{\frac{3}{2}}{\mathbb{R}}) \cdot f))'_{\restriction Z}(x) = (a + x)_{\mathbb{R}}^{\frac{1}{2}}$.

(29) Suppose $Z \subseteq \mathrm{dom}((-\frac{2}{3})\,((\overset{\frac{3}{2}}{\mathbb{R}}) \cdot f))$ and for every x such that $x \in Z$ holds $f(x) = a - x$ and $f(x) > 0$. Then $(-\frac{2}{3})\,((\overset{\frac{3}{2}}{\mathbb{R}}) \cdot f)$ is differentiable on Z and for every x such that $x \in Z$ holds $((-\frac{2}{3})\,((\overset{\frac{3}{2}}{\mathbb{R}}) \cdot f))'_{\restriction Z}(x) = (a - x)_{\mathbb{R}}^{\frac{1}{2}}$.

(30) Suppose $Z \subseteq \mathrm{dom}(2\,((\overset{\frac{1}{2}}{\mathbb{R}}) \cdot f))$ and for every x such that $x \in Z$ holds $f(x) = a + x$ and $f(x) > 0$. Then $2\,((\overset{\frac{1}{2}}{\mathbb{R}}) \cdot f)$ is differentiable on Z and for every x such that $x \in Z$ holds $(2\,((\overset{\frac{1}{2}}{\mathbb{R}}) \cdot f))'_{\restriction Z}(x) = (a + x)_{\mathbb{R}}^{-\frac{1}{2}}$.

(31) Suppose $Z \subseteq \mathrm{dom}((-2)\,((\overset{\frac{1}{2}}{\mathbb{R}}) \cdot f))$ and for every x such that $x \in Z$ holds $f(x) = a - x$ and $f(x) > 0$. Then $(-2)\,((\overset{\frac{1}{2}}{\mathbb{R}}) \cdot f)$ is differentiable on Z and for every x such that $x \in Z$ holds $((-2)\,((\overset{\frac{1}{2}}{\mathbb{R}}) \cdot f))'_{\restriction Z}(x) = (a - x)_{\mathbb{R}}^{-\frac{1}{2}}$.

(32) Suppose $Z \subseteq \mathrm{dom}(\frac{2}{3\cdot b}\,((\overset{\frac{3}{2}}{\mathbb{R}}) \cdot f))$ and for every x such that $x \in Z$ holds $f(x) = a + b \cdot x$ and $b \neq 0$ and $f(x) > 0$. Then $\frac{2}{3\cdot b}\,((\overset{\frac{3}{2}}{\mathbb{R}}) \cdot f)$ is differentiable on Z and for every x such that $x \in Z$ holds $(\frac{2}{3\cdot b}\,((\overset{\frac{3}{2}}{\mathbb{R}}) \cdot f))'_{\restriction Z}(x) = (a+b\cdot x)_{\mathbb{R}}^{\frac{1}{2}}$.

(33) Suppose $Z \subseteq \mathrm{dom}((-\frac{2}{3\cdot b})\,((\overset{\frac{3}{2}}{\mathbb{R}}) \cdot f))$ and for every x such that $x \in Z$ holds $f(x) = a - b \cdot x$ and $b \neq 0$ and $f(x) > 0$. Then $(-\frac{2}{3\cdot b})\,((\overset{\frac{3}{2}}{\mathbb{R}}) \cdot f)$ is differentiable on Z and for every x such that $x \in Z$ holds $((-\frac{2}{3\cdot b})\,((\overset{\frac{3}{2}}{\mathbb{R}}) \cdot f))'_{\restriction Z}(x) = (a - b \cdot x)_{\mathbb{R}}^{\frac{1}{2}}$.

(34) Suppose $Z \subseteq \mathrm{dom}((\overset{\frac{1}{2}}{\mathbb{R}}) \cdot f)$ and $f = f_1 + f_2$ and $f_2 = \frac{2}{\mathbb{Z}}$ and for every x such that $x \in Z$ holds $f_1(x) = a^{\mathbf{2}}$ and $f(x) > 0$. Then $(\overset{\frac{1}{2}}{\mathbb{R}}) \cdot f$ is differentiable on

Z and for every x such that $x \in Z$ holds $((\frac{1}{2}_{\mathbb{R}}) \cdot f)'_{\restriction Z}(x) = x \cdot (a^{\mathbf{2}} + x^2)_{\mathbb{R}}^{-\frac{1}{2}}$.

(35) Suppose $Z \subseteq \mathrm{dom}(-(\frac{1}{2}_{\mathbb{R}}) \cdot f)$ and $f = f_1 - f_2$ and $f_2 = \frac{2}{\mathbb{Z}}$ and for every x such that $x \in Z$ holds $f_1(x) = a^{\mathbf{2}}$ and $f(x) > 0$. Then $-(\frac{1}{2}_{\mathbb{R}}) \cdot f$ is differentiable on Z and for every x such that $x \in Z$ holds $(-(\frac{1}{2}_{\mathbb{R}}) \cdot f)'_{\restriction Z}(x) = x \cdot (a^{\mathbf{2}} - x^2)_{\mathbb{R}}^{-\frac{1}{2}}$.

(36) Suppose $Z \subseteq \mathrm{dom}(2\,((\frac{1}{2}_{\mathbb{R}}) \cdot f))$ and $f = f_1 + f_2$ and $f_2 = \frac{2}{\mathbb{Z}}$ and for every x such that $x \in Z$ holds $f_1(x) = x$ and $f(x) > 0$. Then $2\,((\frac{1}{2}_{\mathbb{R}}) \cdot f)$ is differentiable on Z and for every x such that $x \in Z$ holds $(2\,((\frac{1}{2}_{\mathbb{R}}) \cdot f))'_{\restriction Z}(x) = (2 \cdot x + 1) \cdot (x^2 + x)_{\mathbb{R}}^{-\frac{1}{2}}$.

(37) Suppose $Z \subseteq \mathrm{dom}(($the function sin$) \cdot f)$ and for every x such that $x \in Z$ holds $f(x) = a \cdot x + b$. Then

(i) (the function sin) $\cdot f$ is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $(($the function sin$) \cdot f)'_{\restriction Z}(x) = a \cdot ($the function cos$)(a \cdot x + b)$.

(38) Suppose $Z \subseteq \mathrm{dom}(($the function cos$) \cdot f)$ and for every x such that $x \in Z$ holds $f(x) = a \cdot x + b$. Then

(i) (the function cos) $\cdot f$ is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $(($the function cos$) \cdot f)'_{\restriction Z}(x) = -a \cdot ($the function sin$)(a \cdot x + b)$.

(39) Suppose that for every x such that $x \in Z$ holds (the function cos)$(x) \neq 0$. Then

(i) $\frac{1}{\text{the function cos}}$ is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $(\frac{1}{\text{the function cos}})'_{\restriction Z}(x) = \frac{(\text{the function sin})(x)}{(\text{the function cos})(x)^{\mathbf{2}}}$.

(40) Suppose that for every x such that $x \in Z$ holds (the function sin)$(x) \neq 0$. Then

(i) $\frac{1}{\text{the function sin}}$ is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $(\frac{1}{\text{the function sin}})'_{\restriction Z}(x) = -\frac{(\text{the function cos})(x)}{(\text{the function sin})(x)^{\mathbf{2}}}$.

(41) Suppose $Z \subseteq \mathrm{dom}(($the function sin$)$ $($the function cos$))$. Then

(i) (the function sin) (the function cos) is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $(($the function sin$)$ $($the function cos$))'_{\restriction Z}(x) = \cos(2 \cdot x)$.

(42) Suppose $Z \subseteq \mathrm{dom}(\log_(e) \cdot ($the function cos$))$ and for every x such that $x \in Z$ holds (the function cos)$(x) > 0$. Then $\log_(e) \cdot ($the function cos$)$ is differentiable on Z and for every x such that $x \in Z$ holds $(\log_(e) \cdot ($the function cos$))'_{\restriction Z}(x) = -\tan x$.

(43) Suppose $Z \subseteq \mathrm{dom}(\log\_(e) \cdot (\text{the function sin}))$ and for every x such that $x \in Z$ holds (the function sin)$(x) > 0$. Then $\log\_(e) \cdot$ (the function sin) is differentiable on Z and for every x such that $x \in Z$ holds $(\log\_(e) \cdot (\text{the function sin}))'_{\restriction Z}(x) = \cot x$.

(44) Suppose $Z \subseteq \mathrm{dom}((-\mathrm{id}_Z)\,(\text{the function cos}))$. Then

(i) $(-\mathrm{id}_Z)$ (the function cos) is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $((-\mathrm{id}_Z)\,(\text{the function cos}))'_{\restriction Z}(x) = -(\text{the function cos})(x) + x \cdot (\text{the function sin})(x)$.

(45) Suppose $Z \subseteq \mathrm{dom}(\mathrm{id}_Z\,(\text{the function sin}))$. Then

(i) id_Z (the function sin) is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $(\mathrm{id}_Z\,(\text{the function sin}))'_{\restriction Z}(x) = (\text{the function sin})(x) + x \cdot (\text{the function cos})(x)$.

(46) Suppose $Z \subseteq \mathrm{dom}((-\mathrm{id}_Z)\,(\text{the function cos})+\text{the function sin})$. Then

(i) $(-\mathrm{id}_Z)$ (the function cos)+the function sin is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $((-\mathrm{id}_Z)\,(\text{the function cos})+\text{the function sin})'_{\restriction Z}(x) = x \cdot (\text{the function sin})(x)$.

(47) Suppose $Z \subseteq \mathrm{dom}(\mathrm{id}_Z\,(\text{the function sin})+\text{the function cos})$. Then

(i) id_Z (the function sin)+the function cos is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $(\mathrm{id}_Z\,(\text{the function sin})+\text{the function cos})'_{\restriction Z}(x) = x \cdot (\text{the function cos})(x)$.

(48) Suppose $Z \subseteq \mathrm{dom}(2\,((\frac{\frac{1}{2}}{\mathbb{R}}) \cdot (\text{the function sin})))$ and for every x such that $x \in Z$ holds (the function sin)$(x) > 0$. Then

(i) $2\,((\frac{\frac{1}{2}}{\mathbb{R}}) \cdot (\text{the function sin}))$ is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $(2\,((\frac{\frac{1}{2}}{\mathbb{R}}) \cdot (\text{the function sin})))'_{\restriction Z}(x) = (\text{the function cos})(x) \cdot (\text{the function sin})(x)_{\mathbb{R}}^{-\frac{1}{2}}$.

(49) Suppose $Z \subseteq \mathrm{dom}(\frac{1}{2}\,((\frac{2}{\mathbb{Z}}) \cdot (\text{the function sin})))$. Then

(i) $\frac{1}{2}\,((\frac{2}{\mathbb{Z}}) \cdot (\text{the function sin}))$ is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $(\frac{1}{2}\,((\frac{2}{\mathbb{Z}}) \cdot (\text{the function sin})))'_{\restriction Z}(x) = (\text{the function sin})(x) \cdot (\text{the function cos})(x)$.

(50) Suppose that

(i) $Z \subseteq \mathrm{dom}((\text{the function sin})+\frac{1}{2}\,((\frac{2}{\mathbb{Z}}) \cdot (\text{the function sin})))$, and

(ii) for every x such that $x \in Z$ holds (the function sin)$(x) > 0$ and (the function sin)$(x) < 1$.

Then

(iii) $(\text{the function sin})+\frac{1}{2}\,((\frac{2}{\mathbb{Z}}) \cdot (\text{the function sin}))$ is differentiable on Z, and

(iv) for every x such that $x \in Z$ holds $((\text{the function sin})+\frac{1}{2}\,((\frac{2}{\mathbb{Z}}) \cdot (\text{the function sin})))'_{\restriction Z}(x) = \frac{(\text{the function cos})(x)^3}{1-(\text{the function sin})(x)}$.

(51) Suppose that

(i) $Z \subseteq \mathrm{dom}(\frac{1}{2}\,((\frac{2}{\mathbb{Z}}) \cdot (\text{the function sin}))-\text{the function cos})$, and

(ii) for every x such that $x \in Z$ holds (the function sin)$(x) > 0$ and (the function cos)$(x) < 1$.

Then

(iii) $\frac{1}{2}$ $((\frac{2}{\mathbb{Z}})$ $\cdot$ (the function sin))$-$the function cos is differentiable on Z, and

(iv) for every x such that $x \in Z$ holds ($\frac{1}{2}$ $((\frac{2}{\mathbb{Z}})$ $\cdot$ (the function sin))$-$the function cos$)'_{\restriction Z}(x) = \frac{\text{(the function sin)}(x)^3}{1-\text{(the function cos)}(x)}$.

(52) Suppose that

(i) $Z \subseteq$ dom((the function sin)$-\frac{1}{2}$ $((\frac{2}{\mathbb{Z}})$ $\cdot$ (the function sin))), and

(ii) for every x such that $x \in Z$ holds (the function sin)$(x) > 0$ and (the function sin)$(x) > -1$.

Then

(iii) (the function sin)$-\frac{1}{2}$ $((\frac{2}{\mathbb{Z}})$ $\cdot$ (the function sin)) is differentiable on Z, and

(iv) for every x such that $x \in Z$ holds ((the function sin)$-\frac{1}{2}$ $((\frac{2}{\mathbb{Z}})$ $\cdot$ (the function sin)))$'_{\restriction Z}(x) = \frac{\text{(the function cos)}(x)^3}{1+\text{(the function sin)}(x)}$.

(53) Suppose that

(i) $Z \subseteq$ dom($-$the function cos $-$ $\frac{1}{2}$ $((\frac{2}{\mathbb{Z}})$ $\cdot$ (the function sin))), and

(ii) for every x such that $x \in Z$ holds (the function sin)$(x) > 0$ and (the function cos)$(x) > -1$.

Then

(iii) $-$the function cos $-$ $\frac{1}{2}$ $((\frac{2}{\mathbb{Z}})$ $\cdot$ (the function sin)) is differentiable on Z, and

(iv) for every x such that $x \in Z$ holds ($-$the function cos $-$ $\frac{1}{2}$ $((\frac{2}{\mathbb{Z}})$ $\cdot$ (the function sin)))$'_{\restriction Z}(x) = \frac{\text{(the function sin)}(x)^3}{1+\text{(the function cos)}(x)}$.

(54) Suppose $Z \subseteq$ dom($\frac{1}{n}$ $((\frac{n}{\mathbb{Z}})$ $\cdot$ (the function sin))) and $n > 0$. Then

(i) $\frac{1}{n}$ $((\frac{n}{\mathbb{Z}})$ $\cdot$ (the function sin)) is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds ($\frac{1}{n}$ $((\frac{n}{\mathbb{Z}})$ $\cdot$ (the function sin)))$'_{\restriction Z}(x) =$ ((the function sin)$(x)^{n-1}_{\mathbb{Z}}$) $\cdot$ (the function cos)(x).

(55) Suppose $Z \subseteq \mathrm{dom}(\exp f)$ and for every x such that $x \in Z$ holds $f(x) = x - 1$. Then $\exp f$ is differentiable on Z and for every x such that $x \in Z$ holds $(\exp f)'_{\restriction Z}(x) = x \cdot \exp(x)$.

(56) Suppose $Z \subseteq \mathrm{dom}(\log\_(e) \cdot \frac{\exp}{\exp + f})$ and for every x such that $x \in Z$ holds $f(x) = 1$. Then $\log\_(e) \cdot \frac{\exp}{\exp + f}$ is differentiable on Z and for every x such that $x \in Z$ holds $(\log\_(e) \cdot \frac{\exp}{\exp + f})'_{\restriction Z}(x) = \frac{1}{\exp(x)+1}$.

(57) Suppose $Z \subseteq \mathrm{dom}(\log\_(e) \cdot \frac{\exp - f}{\exp})$ and for every x such that $x \in Z$ holds $f(x) = 1$ and $(\exp - f)(x) > 0$. Then $\log\_(e) \cdot \frac{\exp - f}{\exp}$ is differentiable on Z and for every x such that $x \in Z$ holds $(\log\_(e) \cdot \frac{\exp - f}{\exp})'_{\restriction Z}(x) = \frac{1}{\exp(x)-1}$.

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