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Predicate Calculus for Boolean Valued Functions. Part IX

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Summary. In this paper, we proved some elementary predicate calculus formulae containing the quantifiers of Boolean valued functions with respect to partitions. Such a theory is an analogy of usual predicate logic.

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The terminology and notation used in this paper are introduced in the following papers: [1], [2], [3], [4], and [5].

In this paper Y is a non empty set.

The following propositions are true:

(1) Let a be an element of $\mathrm{BVF}(Y)$, G be a subset of $\mathrm{PARTITIONS}(Y)$, and A, B, C be partitions of Y. Suppose G is a coordinate and $G = \{A, B, C\}$ and $A \neq B$ and $B \neq C$ and $C \neq A$. Then $\neg\exists_{\exists_{a,A}G,B}G \Subset \neg\exists_{\exists_{a,B}G,A}G$.

(2) Let a be an element of $\mathrm{BVF}(Y)$, G be a subset of $\mathrm{PARTITIONS}(Y)$, and A, B, C be partitions of Y. Suppose G is a coordinate and $G = \{A, B, C\}$ and $A \neq B$ and $B \neq C$ and $C \neq A$. Then $\neg\forall_{\forall_{a,A}G,B}G \Subset \neg\forall_{\forall_{a,B}G,A}G$.

(3) Let a be an element of $\mathrm{BVF}(Y)$, G be a subset of $\mathrm{PARTITIONS}(Y)$, and A, B, C be partitions of Y. Suppose G is a coordinate and $G = \{A, B, C\}$ and $A \neq B$ and $B \neq C$ and $C \neq A$. Then $\neg\exists_{\forall_{a,A}G,B}G \Subset \neg\forall_{\forall_{a,B}G,A}G$.

(4) Let a be an element of $\mathrm{BVF}(Y)$, G be a subset of $\mathrm{PARTITIONS}(Y)$, and A, B, C be partitions of Y. Suppose G is a coordinate and $G = \{A, B, C\}$ and $A \neq B$ and $B \neq C$ and $C \neq A$. Then $\neg\forall_{\exists_{a,A}G,B}G \Subset \neg\forall_{\forall_{a,B}G,A}G$.

(5) Let a be an element of $\mathrm{BVF}(Y)$, G be a subset of $\mathrm{PARTITIONS}(Y)$, and A, B, C be partitions of Y. Suppose G is a coordinate and $G = \{A, B, C\}$ and $A \neq B$ and $B \neq C$ and $C \neq A$. Then $\neg\exists_{\exists_{a,A}G,B}G \Subset \neg\forall_{\forall_{a,B}G,A}G$.

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(6) Let a be an element of $\mathrm{BVF}(Y)$, G be a subset of $\mathrm{PARTITIONS}(Y)$, and A, B, C be partitions of Y. Suppose G is a coordinate and $G = \{A, B, C\}$ and $A \neq B$ and $B \neq C$ and $C \neq A$. Then $\neg \exists_{\forall_{a,A}G,B}G \Subset \exists_{\neg \forall_{a,B}G,A}G$.

(7) Let a be an element of $\mathrm{BVF}(Y)$, G be a subset of $\mathrm{PARTITIONS}(Y)$, and A, B, C be partitions of Y. Suppose G is a coordinate and $G = \{A, B, C\}$ and $A \neq B$ and $B \neq C$ and $C \neq A$. Then $\neg \forall_{\exists_{a,A}G,B}G \Subset \exists_{\neg \forall_{a,B}G,A}G$.

(8) Let a be an element of $\mathrm{BVF}(Y)$, G be a subset of $\mathrm{PARTITIONS}(Y)$, and A, B, C be partitions of Y. Suppose G is a coordinate and $G = \{A, B, C\}$ and $A \neq B$ and $B \neq C$ and $C \neq A$. Then $\neg \exists_{\exists_{a,A}G,B}G \Subset \exists_{\neg \forall_{a,B}G,A}G$.

(9) Let a be an element of $\mathrm{BVF}(Y)$, G be a subset of $\mathrm{PARTITIONS}(Y)$, and A, B, C be partitions of Y. Suppose G is a coordinate and $G = \{A, B, C\}$ and $A \neq B$ and $B \neq C$ and $C \neq A$. Then $\neg \forall_{\exists_{a,A}G,B}G \Subset \forall_{\neg \forall_{a,B}G,A}G$.

(10) Let a be an element of $\mathrm{BVF}(Y)$, G be a subset of $\mathrm{PARTITIONS}(Y)$, and A, B, C be partitions of Y. Suppose G is a coordinate and $G = \{A, B, C\}$ and $A \neq B$ and $B \neq C$ and $C \neq A$. Then $\neg \exists_{\exists_{a,A}G,B}G \Subset \forall_{\neg \forall_{a,B}G,A}G$.

(11) Let a be an element of $\mathrm{BVF}(Y)$, G be a subset of $\mathrm{PARTITIONS}(Y)$, and A, B, C be partitions of Y. Suppose G is a coordinate and $G = \{A, B, C\}$ and $A \neq B$ and $B \neq C$ and $C \neq A$. Then $\neg \exists_{\exists_{a,A}G,B}G \Subset \exists_{\neg \exists_{a,B}G,A}G$.

(12) Let a be an element of $\mathrm{BVF}(Y)$, G be a subset of $\mathrm{PARTITIONS}(Y)$, and A, B, C be partitions of Y. Suppose G is a coordinate and $G = \{A, B, C\}$ and $A \neq B$ and $B \neq C$ and $C \neq A$. Then $\neg \exists_{\exists_{a,A}G,B}G \Subset \forall_{\neg \exists_{a,B}G,A}G$.

(13) Let a be an element of $\mathrm{BVF}(Y)$, G be a subset of $\mathrm{PARTITIONS}(Y)$, and A, B, C be partitions of Y. Suppose G is a coordinate and $G = \{A, B, C\}$ and $A \neq B$ and $B \neq C$ and $C \neq A$. Then $\neg \forall_{\exists_{a,A}G,B}G \Subset \exists_{\exists_{\neg a,B}G,A}G$.

(14) Let a be an element of $\mathrm{BVF}(Y)$, G be a subset of $\mathrm{PARTITIONS}(Y)$, and A, B, C be partitions of Y. Suppose G is a coordinate and $G = \{A, B, C\}$ and $A \neq B$ and $B \neq C$ and $C \neq A$. Then $\neg \exists_{\exists_{a,A}G,B}G \Subset \exists_{\exists_{\neg a,B}G,A}G$.

(15) Let a be an element of $\mathrm{BVF}(Y)$, G be a subset of $\mathrm{PARTITIONS}(Y)$, and A, B, C be partitions of Y. Suppose G is a coordinate and $G = \{A, B, C\}$ and $A \neq B$ and $B \neq C$ and $C \neq A$. Then $\neg \forall_{\exists_{a,A}G,B}G \Subset \forall_{\exists_{\neg a,B}G,A}G$.

(16) Let a be an element of $\mathrm{BVF}(Y)$, G be a subset of $\mathrm{PARTITIONS}(Y)$, and A, B, C be partitions of Y. Suppose G is a coordinate and $G = \{A, B, C\}$ and $A \neq B$ and $B \neq C$ and $C \neq A$. Then $\neg \exists_{\exists_{a,A}G,B}G \Subset \forall_{\exists_{\neg a,B}G,A}G$.

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