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# Propositional Calculus for Boolean Valued Functions. Part VI

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**Summary.** In this paper, we proved some elementary propositional calculus formulae for Boolean valued functions.

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The articles [1] and [2] provide the notation and terminology for this paper.

In this paper $Y$ is a non empty set.

The following propositions are true:

(1) For all elements $a$, $b$, $c$ of $\mathrm{BVF}(Y)$ holds $a \wedge b \vee b \wedge c \vee c \wedge a = (a \vee b) \wedge (b \vee c) \wedge (c \vee a)$.

(2) For all elements $a$, $b$, $c$ of $\mathrm{BVF}(Y)$ holds $a \wedge \neg b \vee b \wedge \neg c \vee c \wedge \neg a = b \wedge \neg a \vee c \wedge \neg b \vee a \wedge \neg c$.

(3) For all elements $a$, $b$, $c$ of $\mathrm{BVF}(Y)$ holds $(a \vee \neg b) \wedge (b \vee \neg c) \wedge (c \vee \neg a) = (b \vee \neg a) \wedge (c \vee \neg b) \wedge (a \vee \neg c)$.

(4) For all elements $a$, $b$, $c$ of $\mathrm{BVF}(Y)$ such that $c \Rightarrow a = \mathit{true}(Y)$ and $c \Rightarrow b = \mathit{true}(Y)$ holds $c \Rightarrow a \vee b = \mathit{true}(Y)$.

(5) For all elements $a$, $b$, $c$ of $\mathrm{BVF}(Y)$ such that $a \Rightarrow c = \mathit{true}(Y)$ and $b \Rightarrow c = \mathit{true}(Y)$ holds $a \wedge b \Rightarrow c = \mathit{true}(Y)$.

(6) For all elements $a_1$, $a_2$, $b_1$, $b_2$, $c_1$, $c_2$ of $\mathrm{BVF}(Y)$ holds $(a_1 \Rightarrow a_2) \wedge (b_1 \Rightarrow b_2) \wedge (c_1 \Rightarrow c_2) \wedge (a_1 \vee b_1 \vee c_1) \Subset a_2 \vee b_2 \vee c_2$.

(7) For all elements $a_1$, $a_2$, $b_1$, $b_2$ of $\mathrm{BVF}(Y)$ holds $(a_1 \Rightarrow b_1) \wedge (a_2 \Rightarrow b_2) \wedge (a_1 \vee a_2) \wedge \neg(b_1 \wedge b_2) = (b_1 \Rightarrow a_1) \wedge (b_2 \Rightarrow a_2) \wedge (b_1 \vee b_2) \wedge \neg(a_1 \wedge a_2)$.

(8) For all elements $a$, $b$, $c$, $d$ of $\mathrm{BVF}(Y)$ holds $(a \vee b) \wedge (c \vee d) = a \wedge c \vee a \wedge d \vee b \wedge c \vee b \wedge d$.




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(9) For all elements $a_1$, $a_2$, $b_1$, $b_2$, $b_3$ of $\mathrm{BVF}(Y)$ holds $a_1 \wedge a_2 \vee b_1 \wedge b_2 \wedge b_3 = (a_1 \vee b_1) \wedge (a_1 \vee b_2) \wedge (a_1 \vee b_3) \wedge (a_2 \vee b_1) \wedge (a_2 \vee b_2) \wedge (a_2 \vee b_3)$.

(10) For all elements $a$, $b$, $c$, $d$ of $\mathrm{BVF}(Y)$ holds $(a \Rightarrow b) \wedge (b \Rightarrow c) \wedge (c \Rightarrow d) = (a \Rightarrow b \wedge c \wedge d) \wedge (b \Rightarrow c \wedge d) \wedge (c \Rightarrow d)$.

(11) For all elements $a$, $b$, $c$, $d$ of $\mathrm{BVF}(Y)$ holds $(a \Rightarrow c) \wedge (b \Rightarrow d) \wedge (a \vee b) \Subset c \vee d$.

(12) For all elements $a$, $b$, $c$ of $\mathrm{BVF}(Y)$ holds $(a \wedge b \Rightarrow \neg c) \wedge a \wedge c \Subset \neg b$.

(13) For all elements $a_1$, $a_2$, $a_3$, $b_1$, $b_2$, $b_3$ of $\mathrm{BVF}(Y)$ holds $a_1 \wedge a_2 \wedge a_3 \Rightarrow b_1 \vee b_2 \vee b_3 = \neg b_1 \wedge \neg b_2 \wedge a_3 \Rightarrow \neg a_1 \vee \neg a_2 \vee b_3$.

(14) For all elements $a$, $b$, $c$ of $\mathrm{BVF}(Y)$ holds $(a \Rightarrow b) \wedge (b \Rightarrow c) \wedge (c \Rightarrow a) = a \wedge b \wedge c \vee \neg a \wedge \neg b \wedge \neg c$.

(15) For all elements $a$, $b$, $c$ of $\mathrm{BVF}(Y)$ holds $(a \Rightarrow b) \wedge (b \Rightarrow c) \wedge (c \Rightarrow a) \wedge (a \vee b \vee c) = a \wedge b \wedge c$.

(16) For all elements $a$, $b$, $c$ of $\mathrm{BVF}(Y)$ holds $(a \vee b) \wedge (b \vee c) \wedge (c \vee a) \wedge \neg(a \wedge b \wedge c) = \neg a \wedge b \wedge c \vee a \wedge \neg b \wedge c \vee a \wedge b \wedge \neg c$.

(17) For all elements $a$, $b$, $c$ of $\mathrm{BVF}(Y)$ holds $(a \Rightarrow b) \wedge (b \Rightarrow c) \Subset a \Rightarrow b \wedge c$.

(18) For all elements $a$, $b$, $c$ of $\mathrm{BVF}(Y)$ holds $(a \Rightarrow b) \wedge (b \Rightarrow c) \Subset a \vee b \Rightarrow c$.

(19) For all elements $a$, $b$, $c$ of $\mathrm{BVF}(Y)$ holds $(a \Rightarrow b) \wedge (b \Rightarrow c) \Subset a \Rightarrow b \vee c$.

(20) For all elements $a$, $b$, $c$ of $\mathrm{BVF}(Y)$ holds $(a \Rightarrow b) \wedge (b \Rightarrow c) \Subset a \Rightarrow b \vee \neg c$.

(21) For all elements $a$, $b$, $c$ of $\mathrm{BVF}(Y)$ holds $(a \Rightarrow b) \wedge (b \Rightarrow c) \Subset b \Rightarrow c \vee a$.

(22) For all elements $a$, $b$, $c$ of $\mathrm{BVF}(Y)$ holds $(a \Rightarrow b) \wedge (b \Rightarrow c) \Subset b \Rightarrow c \vee \neg a$.

(23) For all elements $a$, $b$, $c$ of $\mathrm{BVF}(Y)$ holds $(a \Rightarrow b) \wedge (b \Rightarrow c) \Subset (a \Rightarrow b) \wedge (b \Rightarrow c \vee a)$.

(24) For all elements $a$, $b$, $c$ of $\mathrm{BVF}(Y)$ holds $(a \Rightarrow b) \wedge (b \Rightarrow c) \Subset (a \Rightarrow b \vee \neg c) \wedge (b \Rightarrow c)$.

(25) For all elements $a$, $b$, $c$ of $\mathrm{BVF}(Y)$ holds $(a \Rightarrow b) \wedge (b \Rightarrow c) \Subset (a \Rightarrow b \vee c) \wedge (b \Rightarrow c \vee a)$.

(26) For all elements $a$, $b$, $c$ of $\mathrm{BVF}(Y)$ holds $(a \Rightarrow b) \wedge (b \Rightarrow c) \Subset (a \Rightarrow b \vee \neg c) \wedge (b \Rightarrow c \vee a)$.

(27) For all elements $a$, $b$, $c$ of $\mathrm{BVF}(Y)$ holds $(a \Rightarrow b) \wedge (b \Rightarrow c) \Subset (a \Rightarrow b \vee \neg c) \wedge (b \Rightarrow c \vee \neg a)$.

## References

[1] Shunichi Kobayashi and Kui Jia. A theory of Boolean valued functions and partitions. *Formalized Mathematics*, 7(**2**):249–254, 1998.

[2] Zinaida Trybulec. Properties of subsets. *Formalized Mathematics*, 1(**1**):67–71, 1990.

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