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Binary Arithmetics. Binary Sequences

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The notation and terminology used here are introduced in the following papers: [10], [9], [7], [3], [2], [4], [12], [6], [5], [14], [1], [8], [15], [11], and [13].

1. Binary Arithmetics

The following propositions are true:

(1) For every non empty natural number n and for every tuple F of n and *Boolean* holds $\mathrm{Absval}(F) <$ the n-th power of 2.

(2) For every non empty natural number n and for all tuples F_1, F_2 of n and *Boolean* such that $\mathrm{Absval}(F_1) = \mathrm{Absval}(F_2)$ holds $F_1 = F_2$.

(3) For all finite sequences t_1, t_2 such that $\mathrm{Rev}(t_1) = \mathrm{Rev}(t_2)$ holds $t_1 = t_2$.

(4) For every natural number n holds $\langle\underbrace{0, \ldots, 0}_{n+1}\rangle = \langle\underbrace{0, \ldots, 0}_{n}\rangle \frown \langle 0\rangle$.

(5) For every natural number n holds $\langle\underbrace{0, \ldots, 0}_{n}\rangle \in \mathit{Boolean}^*$.

(6) For every natural number n and for every tuple y of n and *Boolean* such that $y = \langle\underbrace{0, \ldots, 0}_{n}\rangle$ holds $\neg y = n \mapsto 1$.

(7) For every non empty natural number n and for every tuple F of n and *Boolean* such that $F = \langle\underbrace{0, \ldots, 0}_{n}\rangle$ holds $\mathrm{Absval}(F) = 0$.

(8) Let n be a non empty natural number and F be a tuple of n and *Boolean*. If $F = \langle\underbrace{0, \ldots, 0}_{n}\rangle$, then $\mathrm{Absval}(\neg F) =$ (the n-th power of 2)-1.

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(9) For every natural number n holds $\mathrm{Rev}(\langle\underbrace{0,\ldots,0}_{n}\rangle) = \langle\underbrace{0,\ldots,0}_{n}\rangle$.

(10) For every natural number n and for every tuple y of n and *Boolean* such that $y = \langle\underbrace{0,\ldots,0}_{n}\rangle$ holds $\mathrm{Rev}(\neg y) = \neg y$.

(11) $\mathrm{Bin1}(1) = \langle \mathit{true}\rangle$.

(12) For every non empty natural number n holds $\mathrm{Absval}(\mathrm{Bin1}(n)) = 1$.

(13) For all elements x, y of *Boolean* holds $x \vee y = \mathit{true}$ iff $x = \mathit{true}$ or $y = \mathit{true}$ and $x \vee y = \mathit{false}$ iff $x = \mathit{false}$ and $y = \mathit{false}$.

(14) For all elements x, y of *Boolean* holds $\mathrm{add\_ovfl}(\langle x\rangle, \langle y\rangle) = \mathit{true}$ iff $x = \mathit{true}$ and $y = \mathit{true}$.

(15) $\neg\langle \mathit{false}\rangle = \langle \mathit{true}\rangle$.

(16) $\neg\langle \mathit{true}\rangle = \langle \mathit{false}\rangle$.

(17) $\langle \mathit{false}\rangle + \langle \mathit{false}\rangle = \langle \mathit{false}\rangle$.

(18) $\langle \mathit{false}\rangle + \langle \mathit{true}\rangle = \langle \mathit{true}\rangle$ and $\langle \mathit{true}\rangle + \langle \mathit{false}\rangle = \langle \mathit{true}\rangle$.

(19) $\langle \mathit{true}\rangle + \langle \mathit{true}\rangle = \langle \mathit{false}\rangle$.

(20) Let n be a non empty natural number and x, y be tuples of n and *Boolean*. Suppose $\pi_n x = \mathit{true}$ and $\pi_n \,\mathrm{carry}(x, \mathrm{Bin1}(n)) = \mathit{true}$. Let k be a non empty natural number. If $k \neq 1$ and $k \leqslant n$, then $\pi_k x = \mathit{true}$ and $\pi_k \,\mathrm{carry}(x, \mathrm{Bin1}(n)) = \mathit{true}$.

(21) For every non empty natural number n and for every tuple x of n and *Boolean* such that $\pi_n x = \mathit{true}$ and $\pi_n \,\mathrm{carry}(x, \mathrm{Bin1}(n)) = \mathit{true}$ holds $\mathrm{carry}(x, \mathrm{Bin1}(n)) = \neg\,\mathrm{Bin1}(n)$.

(22) Let n be a non empty natural number and x, y be tuples of n and *Boolean*. If $y = \langle\underbrace{0,\ldots,0}_{n}\rangle$ and $\pi_n x = \mathit{true}$ and $\pi_n \,\mathrm{carry}(x, \mathrm{Bin1}(n)) = \mathit{true}$, then $x = \neg y$.

(23) For every non empty natural number n and for every tuple y of n and *Boolean* such that $y = \langle\underbrace{0,\ldots,0}_{n}\rangle$ holds $\mathrm{carry}(\neg y, \mathrm{Bin1}(n)) = \neg\,\mathrm{Bin1}(n)$.

(24) Let n be a non empty natural number and x, y be tuples of n and *Boolean*. If $y = \langle\underbrace{0,\ldots,0}_{n}\rangle$, then $\mathrm{add\_ovfl}(x, \mathrm{Bin1}(n)) = \mathit{true}$ iff $x = \neg y$.

(25) For every non empty natural number n and for every tuple z of n and *Boolean* such that $z = \langle\underbrace{0,\ldots,0}_{n}\rangle$ holds $\neg z + \mathrm{Bin1}(n) = z$.

2. Binary Sequences

Let n, k be natural numbers. The functor n-BinarySequence(k) yielding a tuple of n and *Boolean* is defined by:

(Def. 1) For every natural number i such that $i \in \operatorname{Seg} n$ holds $\pi_i(n\text{-BinarySequence}(k)) = ((k \div (\text{the } (i -' 1)\text{-th power of } 2)) \bmod 2 = 0 \to \mathit{false}, \mathit{true})$.

One can prove the following propositions:

(26) For every natural number n holds n-BinarySequence$(0) = \langle\underbrace{0, \ldots, 0}_{n}\rangle$.

(27) For all natural numbers n, k such that $k <$ the n-th power of 2 holds $((n+1)\text{-BinarySequence}(k))(n+1) = \mathit{false}$.

(28) Let n be a non empty natural number and k be a natural number. If $k <$ the n-th power of 2, then $(n+1)$-BinarySequence$(k) = (n\text{-BinarySequence}(k)) \frown \langle \mathit{false} \rangle$.

(29) For every non empty natural number n holds $(n+1)$-BinarySequence(the n-th power of 2) $= \langle\underbrace{0, \ldots, 0}_{n}\rangle \frown \langle \mathit{true} \rangle$.

(30) Let n be a non empty natural number and k be a natural number. Suppose the n-th power of $2 \leqslant k$ and $k <$ the $(n+1)$-th power of 2. Then $((n+1)\text{-BinarySequence}(k))(n+1) = \mathit{true}$.

(31) Let n be a non empty natural number and k be a natural number. Suppose the n-th power of $2 \leqslant k$ and $k <$ the $(n+1)$-th power of 2. Then $(n+1)$-BinarySequence$(k) = (n$-BinarySequence$(k -'$ (the n-th power of 2$)))\frown\langle \mathit{true} \rangle$.

(32) Let n be a non empty natural number and k be a natural number. Suppose $k <$ the n-th power of 2. Let x be a tuple of n and *Boolean*. If $x = \langle\underbrace{0, \ldots, 0}_{n}\rangle$, then n-BinarySequence$(k) = \neg x$ iff $k =$ (the n-th power of 2)-1.

(33) Let n be a non empty natural number and k be a natural number. If $k + 1 <$ the n-th power of 2, then $\text{add\_ovfl}(n\text{-BinarySequence}(k), \text{Bin1}(n)) = \mathit{false}$.

(34) Let n be a non empty natural number and k be a natural number. If $k + 1 <$ the n-th power of 2, then n-BinarySequence$(k+1) = (n\text{-BinarySequence}(k)) + \text{Bin1}(n)$.

(35) For all natural numbers n, i holds $(n+1)$-BinarySequence$(i) = \langle i \bmod 2\rangle \frown (n\text{-BinarySequence}(i \div 2))$.

(36) For every non empty natural number n and for every natural number k

such that $k <$ the n-th power of 2 holds Absval(n-BinarySequence(k)) $=$ k.

(37) For every non empty natural number n and for every tuple x of n and *Boolean* holds n-BinarySequence(Absval(x)) $= x$.

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