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Relocability for $\mathbf{SCM}_{\mathrm{FSA}}$

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The terminology and notation used in this paper are introduced in the following articles: [12], [15], [1], [24], [14], [19], [26], [18], [2], [10], [5], [27], [7], [3], [6], [25], [11], [8], [9], [4], [13], [22], [16], [17], [23], [20], and [21].

1. Relocability

In this paper j, k will denote natural numbers.

Let p be a finite partial state of $\mathbf{SCM}_{\mathrm{FSA}}$ and let k be a natural number. The functor $\mathrm{Relocated}(p,k)$ yields a finite partial state of $\mathbf{SCM}_{\mathrm{FSA}}$ and is defined as follows:

(Def. 1) $\mathrm{Relocated}(p,k) = \mathrm{Start\text{-}At}(\mathbf{IC}_p + k) {+\cdot}\, \mathrm{IncAddr}(\mathrm{Shift}(\mathrm{ProgramPart}(p), k), k) {+\cdot}\, \mathrm{DataPart}(p)$.

We now state a number of propositions:

(1) For every finite partial state p of $\mathbf{SCM}_{\mathrm{FSA}}$ and for every natural number k holds $\mathrm{DataPart}(\mathrm{Relocated}(p,k)) = \mathrm{DataPart}(p)$.

(2) For every finite partial state p of $\mathbf{SCM}_{\mathrm{FSA}}$ and for every natural number k holds $\mathrm{ProgramPart}(\mathrm{Relocated}(p,k)) = \mathrm{IncAddr}(\mathrm{Shift}(\mathrm{ProgramPart}(p), k), k)$.

(3) For every finite partial state p of $\mathbf{SCM}_{\mathrm{FSA}}$ holds $\mathrm{dom}\, \mathrm{ProgramPart}(\mathrm{Relocated}(p,k)) = \{\mathrm{insloc}(j+k) : \mathrm{insloc}(j) \in \mathrm{dom}\, \mathrm{ProgramPart}(p)\}$.

(4) Let p be a finite partial state of $\mathbf{SCM}_{\mathrm{FSA}}$, and let k be a natural number, and let l be an instruction-location of $\mathbf{SCM}_{\mathrm{FSA}}$. Then $l \in \mathrm{dom}\, p$ if and only if $l + k \in \mathrm{dom}\, \mathrm{Relocated}(p,k)$.

(5) For every finite partial state p of $\mathbf{SCM}_{\mathrm{FSA}}$ and for every natural number k holds $\mathbf{IC}_{\mathbf{SCM}_{\mathrm{FSA}}} \in \mathrm{dom}\, \mathrm{Relocated}(p,k)$.

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(6) For every finite partial state p of $\mathbf{SCM}_{\mathrm{FSA}}$ and for every natural number k holds $\mathbf{IC}_{\mathrm{Relocated}(p,k)} = \mathbf{IC}_p + k$.

(7) Let p be a finite partial state of $\mathbf{SCM}_{\mathrm{FSA}}$, and let k be a natural number, and let l_1 be an instruction-location of $\mathbf{SCM}_{\mathrm{FSA}}$, and let I be an instruction of $\mathbf{SCM}_{\mathrm{FSA}}$. If $l_1 \in \mathrm{dom}\,\mathrm{ProgramPart}(p)$ and $I = p(l_1)$, then $\mathrm{IncAddr}(I, k) = (\mathrm{Relocated}(p, k))(l_1 + k)$.

(8) For every finite partial state p of $\mathbf{SCM}_{\mathrm{FSA}}$ and for every natural number k holds $\text{Start-At}(\mathbf{IC}_p + k) \subseteq \mathrm{Relocated}(p, k)$.

(9) Let s be a data-only finite partial state of $\mathbf{SCM}_{\mathrm{FSA}}$, and let p be a finite partial state of $\mathbf{SCM}_{\mathrm{FSA}}$, and let k be a natural number. If $\mathbf{IC}_{\mathbf{SCM}_{\mathrm{FSA}}} \in \mathrm{dom}\,p$, then $\mathrm{Relocated}(p{+}{\cdot}s, k) = \mathrm{Relocated}(p, k){+}{\cdot}s$.

(10) Let k be a natural number, and let p be an autonomic finite partial state of $\mathbf{SCM}_{\mathrm{FSA}}$, and let s_1, s_2 be states of $\mathbf{SCM}_{\mathrm{FSA}}$. If $p \subseteq s_1$ and $\mathrm{Relocated}(p, k) \subseteq s_2$, then $p \subseteq s_1{+}{\cdot}s_2 \restriction (\text{Int-Locations} \cup \text{FinSeq-Locations})$.

2. Main Theorems of Relocability

We now state several propositions:

(11) Let k be a natural number and let p be an autonomic finite partial state of $\mathbf{SCM}_{\mathrm{FSA}}$. Suppose $\mathbf{IC}_{\mathbf{SCM}_{\mathrm{FSA}}} \in \mathrm{dom}\,p$. Let s be a state of $\mathbf{SCM}_{\mathrm{FSA}}$. Suppose $p \subseteq s$. Let i be a natural number. Then $(\mathrm{Computation}(s{+}{\cdot}\,\mathrm{Relocated}(p, k)))(i) = (\mathrm{Computation}(s))(i){+}{\cdot}\,\text{Start-At}(\mathbf{IC}_{(\mathrm{Computation}(s))(i)} + k){+}{\cdot}\,\mathrm{ProgramPart}(\mathrm{Relocated}(p, k))$.

(12) Let k be a natural number, and let p be an autonomic finite partial state of $\mathbf{SCM}_{\mathrm{FSA}}$, and let s_1, s_2, s_3 be states of $\mathbf{SCM}_{\mathrm{FSA}}$. Suppose $\mathbf{IC}_{\mathbf{SCM}_{\mathrm{FSA}}} \in \mathrm{dom}\,p$ and $p \subseteq s_1$ and $\mathrm{Relocated}(p, k) \subseteq s_2$ and $s_3 = s_1{+}{\cdot}s_2 \restriction (\text{Int-Locations} \cup \text{FinSeq-Locations})$. Let i be a natural number. Then $\mathbf{IC}_{(\mathrm{Computation}(s_1))(i)} + k = \mathbf{IC}_{(\mathrm{Computation}(s_2))(i)}$ and $\mathrm{IncAddr}(\mathrm{CurInstr}((\mathrm{Computation}(s_1))(i)), k) = \mathrm{CurInstr}((\mathrm{Computation}(s_2))(i))$ and $(\mathrm{Computation}(s_1))(i) \restriction \mathrm{dom}\,\mathrm{DataPart}(p) = (\mathrm{Computation}(s_2))(i) \restriction \mathrm{dom}\,\mathrm{DataPart}(\mathrm{Relocated}(p, k))$ and $(\mathrm{Computation}(s_3))(i) \restriction (\text{Int-Locations} \cup \text{FinSeq-Locations}) = (\mathrm{Computation}(s_2))(i) \restriction (\text{Int-Locations} \cup \text{FinSeq-Locations})$.

(13) Let p be an autonomic finite partial state of $\mathbf{SCM}_{\mathrm{FSA}}$ and let k be a natural number. If $\mathbf{IC}_{\mathbf{SCM}_{\mathrm{FSA}}} \in \mathrm{dom}\,p$, then p is halting iff $\mathrm{Relocated}(p, k)$ is halting.

(14) Let k be a natural number and let p be an autonomic finite partial state of $\mathbf{SCM}_{\mathrm{FSA}}$. Suppose $\mathbf{IC}_{\mathbf{SCM}_{\mathrm{FSA}}} \in \mathrm{dom}\,p$. Let s be a state of $\mathbf{SCM}_{\mathrm{FSA}}$. Suppose $\mathrm{Relocated}(p, k) \subseteq$

s. Let i be a natural number. Then $(\mathrm{Computation}(s))(i) = (\mathrm{Computation}(s{+\cdot}p))(i){+\cdot}\,\mathrm{Start\text{-}At}(\mathbf{IC}_{(\mathrm{Computation}(s{+\cdot}p))(i)} + k){+\cdot}s \restriction \mathrm{dom}\,\mathrm{ProgramPart}(p){+\cdot}\,\mathrm{ProgramPart}(\mathrm{Relocated}(p,k))$.

(15) Let k be a natural number and let p be a finite partial state of $\mathbf{SCM}_{\mathrm{FSA}}$. Suppose $\mathbf{IC}_{\mathbf{SCM}_{\mathrm{FSA}}} \in \mathrm{dom}\,p$. Let s be a state of $\mathbf{SCM}_{\mathrm{FSA}}$. Suppose $p \subseteq s$ and $\mathrm{Relocated}(p,k)$ is autonomic. Let i be a natural number. Then $(\mathrm{Computation}(s))(i) = (\mathrm{Computation}(s{+\cdot}\,\mathrm{Relocated}(p,k)))(i){+\cdot}\,\mathrm{Start\text{-}At}(\mathbf{IC}_{(\mathrm{Computation}(s{+\cdot}\,\mathrm{Relocated}(p,k)))(i)} -' k){+\cdot}s \restriction \mathrm{dom}\,\mathrm{ProgramPart}(\mathrm{Relocated}(p,k)){+\cdot}\,\mathrm{ProgramPart}(p)$.

(16) Let p be a finite partial state of $\mathbf{SCM}_{\mathrm{FSA}}$. Suppose $\mathbf{IC}_{\mathbf{SCM}_{\mathrm{FSA}}} \in \mathrm{dom}\,p$. Let k be a natural number. Then p is autonomic if and only if $\mathrm{Relocated}(p,k)$ is autonomic.

(17) Let p be a halting autonomic finite partial state of $\mathbf{SCM}_{\mathrm{FSA}}$. If $\mathbf{IC}_{\mathbf{SCM}_{\mathrm{FSA}}} \in \mathrm{dom}\,p$, then for every natural number k holds $\mathrm{DataPart}(\mathrm{Result}(p)) = \mathrm{DataPart}(\mathrm{Result}(\mathrm{Relocated}(p,k)))$.

(18) Let F be a data-only partial function from $\mathrm{FinPartSt}(\mathbf{SCM}_{\mathrm{FSA}})$ to $\mathrm{FinPartSt}(\mathbf{SCM}_{\mathrm{FSA}})$ and let p be a finite partial state of $\mathbf{SCM}_{\mathrm{FSA}}$. Suppose $\mathbf{IC}_{\mathbf{SCM}_{\mathrm{FSA}}} \in \mathrm{dom}\,p$. Let k be a natural number. Then p computes F if and only if $\mathrm{Relocated}(p,k)$ computes F.

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