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Subspaces of Real Linear Space Generated by One, Two, or Three Vectors and Their Cosets

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The articles [7], [2], [1], [3], [4], [11], [10], [5], [6], [9], and [8] provide the notation and terminology for this paper. For simplicity we adopt the following rules: x is arbitrary, a, b, c denote real numbers, V denotes a real linear space, u, v, v_1, v_2, v_3, w, w_1, w_2, w_3 denote vectors of V, and W, W_1, W_2 denote subspaces of V. In this article we present several logical schemes. The scheme *LambdaSep3* deals with a non-empty set $\mathcal{A}$, a non-empty set $\mathcal{B}$, an element $\mathcal{C}$ of $\mathcal{A}$, an element $\mathcal{D}$ of $\mathcal{A}$, an element $\mathcal{E}$ of $\mathcal{A}$, an element $\mathcal{F}$ of $\mathcal{B}$, an element $\mathcal{G}$ of $\mathcal{B}$, an element $\mathcal{H}$ of $\mathcal{B}$, and a unary functor $\mathcal{F}$ yielding an element of $\mathcal{B}$ and states that:

there exists a function f from $\mathcal{A}$ into $\mathcal{B}$ such that $f(\mathcal{C}) = \mathcal{F}$ and $f(\mathcal{D}) = \mathcal{G}$ and $f(\mathcal{E}) = \mathcal{H}$ and for every element C of $\mathcal{A}$ such that $C \neq \mathcal{C}$ and $C \neq \mathcal{D}$ and $C \neq \mathcal{E}$ holds $f(C) = \mathcal{F}(C)$

provided the parameters have the following properties:

- $\mathcal{C} \neq \mathcal{D}$,
- $\mathcal{C} \neq \mathcal{E}$,
- $\mathcal{D} \neq \mathcal{E}$.

The scheme *LinCEx1* deals with a real linear space $\mathcal{A}$, a vector $\mathcal{B}$ of $\mathcal{A}$, and a real number $\mathcal{C}$ and states that:

there exists a linear combination l of $\{\mathcal{B}\}$ such that $l(\mathcal{B}) = \mathcal{C}$

for all values of the parameters.

The scheme *LinCEx2* deals with a real linear space $\mathcal{A}$, a vector $\mathcal{B}$ of $\mathcal{A}$, a vector $\mathcal{C}$ of $\mathcal{A}$, a real number $\mathcal{D}$, and a real number $\mathcal{E}$ and states that:

there exists a linear combination l of $\{\mathcal{B}, \mathcal{C}\}$ such that $l(\mathcal{B}) = \mathcal{D}$ and $l(\mathcal{C}) = \mathcal{E}$

provided the following condition is satisfied:

- $\mathcal{B} \neq \mathcal{C}$.

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The scheme *LinCEx3* deals with a real linear space $\mathcal{A}$, a vector $\mathcal{B}$ of $\mathcal{A}$, a vector $\mathcal{C}$ of $\mathcal{A}$, a vector $\mathcal{D}$ of $\mathcal{A}$, a real number $\mathcal{E}$, a real number $\mathcal{F}$, and a real number $\mathcal{G}$ and states that:

there exists a linear combination l of $\{\mathcal{B}, \mathcal{C}, \mathcal{D}\}$ such that $l(\mathcal{B}) = \mathcal{E}$ and $l(\mathcal{C}) = \mathcal{F}$ and $l(\mathcal{D}) = \mathcal{G}$

provided the parameters meet the following conditions:

- $\mathcal{B} \neq \mathcal{C}$,
- $\mathcal{B} \neq \mathcal{D}$,
- $\mathcal{C} \neq \mathcal{D}$.

We now state a number of propositions:

(1) $(v + w) - v = w$ and $(w + v) - v = w$ and $(v - v) + w = w$ and $(w-v)+v = w$ and $v+(w-v) = w$ and $w+(v-v) = w$ and $v-(v-w) = w$.

(2) $(v + u) - w = (v - w) + u$.

(3) If $v_1 + w = v_2 + w$, then $v_1 = v_2$.

(4) If $v_1 - w = v_2 - w$, then $v_1 = v_2$.

(5) $v = v_1 + v_2$ if and only if $v_2 = v - v_1$.

(6) $-a \cdot v = (-a) \cdot v$.

(7) If W_1 is a subspace of W_2, then $v + W_1 \subseteq v + W_2$.

(8) If $u \in v + W$, then $v + W = u + W$.

(9) For every linear combination l of $\{u, v, w\}$ such that $u \neq v$ and $u \neq w$ and $v \neq w$ holds $\sum l = l(u) \cdot u + l(v) \cdot v + l(w) \cdot w$.

(10) $u \neq v$ and $u \neq w$ and $v \neq w$ and $\{u, v, w\}$ is linearly independent if and only if for all a, b, c such that $a \cdot u + b \cdot v + c \cdot w = 0_V$ holds $a = 0$ and $b = 0$ and $c = 0$.

(11) $x \in \mathrm{Lin}(\{v\})$ if and only if there exists a such that $x = a \cdot v$.

(12) $v \in \mathrm{Lin}(\{v\})$.

(13) $x \in v + \mathrm{Lin}(\{w\})$ if and only if there exists a such that $x = v + a \cdot w$.

(14) $x \in \mathrm{Lin}(\{w_1, w_2\})$ if and only if there exist a, b such that $x = a \cdot w_1 + b \cdot w_2$.

(15) $w_1 \in \mathrm{Lin}(\{w_1, w_2\})$ and $w_2 \in \mathrm{Lin}(\{w_1, w_2\})$.

(16) $x \in v + \mathrm{Lin}(\{w_1, w_2\})$ if and only if there exist a, b such that $x = v + a \cdot w_1 + b \cdot w_2$.

(17) $x \in \mathrm{Lin}(\{v_1, v_2, v_3\})$ if and only if there exist a, b, c such that $x = a \cdot v_1 + b \cdot v_2 + c \cdot v_3$.

(18) $w_1 \in \mathrm{Lin}(\{w_1, w_2, w_3\})$ and $w_2 \in \mathrm{Lin}(\{w_1, w_2, w_3\})$ and $w_3 \in \mathrm{Lin}(\{w_1, w_2, w_3\})$.

(19) $x \in v + \mathrm{Lin}(\{w_1, w_2, w_3\})$ if and only if there exist a, b, c such that $x = v + a \cdot w_1 + b \cdot w_2 + c \cdot w_3$.

(20) If $\{u, v\}$ is linearly independent and $u \neq v$, then $\{u, v - u\}$ is linearly independent.

(21) If $\{u, v\}$ is linearly independent and $u \neq v$, then $\{u, v + u\}$ is linearly independent.

(22) If $\{u, v\}$ is linearly independent and $u \neq v$ and $a \neq 0$, then $\{u, a \cdot v\}$ is linearly independent.

(23) If $\{u, v\}$ is linearly independent and $u \neq v$, then $\{u, -v\}$ is linearly independent.

(24) If $a \neq b$, then $\{a \cdot v, b \cdot v\}$ is linearly dependent.

(25) If $a \neq 1$, then $\{v, a \cdot v\}$ is linearly dependent.

(26) If $\{u, w, v\}$ is linearly independent and $u \neq v$ and $u \neq w$ and $v \neq w$, then $\{u, w, v - u\}$ is linearly independent.

(27) If $\{u, w, v\}$ is linearly independent and $u \neq v$ and $u \neq w$ and $v \neq w$, then $\{u, w - u, v - u\}$ is linearly independent.

(28) If $\{u, w, v\}$ is linearly independent and $u \neq v$ and $u \neq w$ and $v \neq w$, then $\{u, w, v + u\}$ is linearly independent.

(29) If $\{u, w, v\}$ is linearly independent and $u \neq v$ and $u \neq w$ and $v \neq w$, then $\{u, w + u, v + u\}$ is linearly independent.

(30) If $\{u, w, v\}$ is linearly independent and $u \neq v$ and $u \neq w$ and $v \neq w$ and $a \neq 0$, then $\{u, w, a \cdot v\}$ is linearly independent.

(31) If $\{u, w, v\}$ is linearly independent and $u \neq v$ and $u \neq w$ and $v \neq w$ and $a \neq 0$ and $b \neq 0$, then $\{u, a \cdot w, b \cdot v\}$ is linearly independent.

The following propositions are true:

(32) If $\{u, w, v\}$ is linearly independent and $u \neq v$ and $u \neq w$ and $v \neq w$, then $\{u, w, -v\}$ is linearly independent.

(33) If $\{u, w, v\}$ is linearly independent and $u \neq v$ and $u \neq w$ and $v \neq w$, then $\{u, -w, -v\}$ is linearly independent.

(34) If $a \neq b$, then $\{a \cdot v, b \cdot v, w\}$ is linearly dependent.

(35) If $a \neq 1$, then $\{v, a \cdot v, w\}$ is linearly dependent.

(36) If $v \in \mathrm{Lin}(\{w\})$ and $v \neq 0_V$, then $\mathrm{Lin}(\{v\}) = \mathrm{Lin}(\{w\})$.

(37) If $v_1 \neq v_2$ and $\{v_1, v_2\}$ is linearly independent and $v_1 \in \mathrm{Lin}(\{w_1, w_2\})$ and $v_2 \in \mathrm{Lin}(\{w_1, w_2\})$, then $\mathrm{Lin}(\{w_1, w_2\}) = \mathrm{Lin}(\{v_1, v_2\})$ and $\{w_1, w_2\}$ is linearly independent and $w_1 \neq w_2$.

(38) If $w \neq 0_V$ and $\{v, w\}$ is linearly dependent, then there exists a such that $v = a \cdot w$.

(39) If $v \neq w$ and $\{v, w\}$ is linearly independent and $\{u, v, w\}$ is linearly dependent, then there exist a, b such that $u = a \cdot v + b \cdot w$.

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