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Similarity of Formulae

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Summary. The main objective of the paper is to define the concept of the similarity of formulas. We mean by similar formulas the two formulas that differs only in the names of bound variables. Some authors (compare [16]) call such formulas *congruent*. We use the word *similar* following [14,12,15]. The concept is unjustfully neglected in many logical handbooks. It is intuitively quite clear, however the exact definition is not simple. As far as we know, only W.A.Pogorzelski and T.Prucnal [15] define it in the precise way. We follow basically the Pogorzelski's definition (compare [14]). We define renumaration of bound variables and we say that two formulas are similar if after renumaration are equal. Therefore we need a rule of chosing bound variables independent of the original choice. Quite obvious solution is to use consecutively variables $x_{k+1}, x_{k+2}, \ldots$, where k is the maximal index of free variable occurring in the formula. Therefore after the renumaration we get the new formula in which different quantifiers bind different variables. It is the reason that the result of renumaration applied to a formula φ we call φ *with variables separated.*

MML Identifier: `CQC_SIM1`.

The notation and terminology used in this paper are introduced in the following articles: [23], [27], [20], [24], [19], [13], [5], [6], [18], [3], [10], [26], [21], [11], [2], [25], [22], [8], [17], [1], [9], [4], and [7]. One can prove the following four propositions:

(1) For arbitrary x, y and for every function f holds $(f +\cdot (\{x\} \longmapsto y)) \circ \{x\} = \{y\}$.

(2) For all sets K, L and for arbitrary x, y and for every function f holds $(f +\cdot (L \longmapsto y)) \circ K \subseteq f \circ K \cup \{y\}$.

(3) For arbitrary x, y and for every function g and for every set A holds $(g +\cdot (\{x\} \longmapsto y)) \circ (A \setminus \{x\}) = g \circ (A \setminus \{x\})$.

(4) For arbitrary x, y and for every function g and for every set A such that $y \notin g \circ (A \setminus \{x\})$ holds $(g +\cdot (\{x\} \longmapsto y)) \circ (A \setminus \{x\}) = (g +\cdot (\{x\} \longmapsto y)) \circ A \setminus \{y\}$.

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For simplicity we follow the rules: p, q, r, s denote elements of CQC-WFF, x denotes an element of BoundVar, i, k, l, m, n denote elements of $\mathbb{N}$, l_1 denotes a variables list of k, and P denotes a k-ary predicate symbol. The following propositions are true:

(5) If p is atomic, then there exist k, P, l_1 such that $p = P[l_1]$.

(6) If p is negative, then there exists q such that $p = \neg q$.

(7) If p is conjunctive, then there exist q, r such that $p = q \wedge r$.

(8) If p is universal, then there exist x, q such that $p = \forall_x q$.

(9) For every non-empty set D and for every finite sequence l of elements of D holds $\operatorname{rng} l = \{l(i) : 1 \leq i \wedge i \leq \operatorname{len} l\}$.

In this article we present several logical schemes. The scheme *NUBFuncExD* deals with a non-empty set $\mathcal{A}$, a non-empty set $\mathcal{B}$, and a binary predicate $\mathcal{P}$, and states that:

there exists a function f from $\mathcal{A}$ into $\mathcal{B}$ such that for every element e of $\mathcal{A}$ holds $\mathcal{P}[e, f(e)]$

provided the parameters satisfy the following condition:

- for every element e of $\mathcal{A}$ there exists an element u of $\mathcal{B}$ such that $\mathcal{P}[e, u]$.

The scheme *NUBFuncEx2D* deals with a non-empty set $\mathcal{A}$, a non-empty set $\mathcal{B}$, a non-empty set $\mathcal{C}$, and a ternary predicate $\mathcal{P}$, and states that:

there exists a function f from $[\!: \mathcal{A}, \mathcal{B} :\!]$ into $\mathcal{C}$ such that for every element x of $\mathcal{A}$ and for every element y of $\mathcal{B}$ holds $\mathcal{P}[x, y, f(\langle x, y \rangle)]$

provided the parameters meet the following condition:

- for every element x of $\mathcal{A}$ and for every element y of $\mathcal{B}$ there exists an element u of $\mathcal{C}$ such that $\mathcal{P}[x, y, u]$.

The scheme *QC_Func_ExN* deals with a non-empty set $\mathcal{A}$, an element $\mathcal{B}$ of $\mathcal{A}$, a unary functor $\mathcal{F}$ yielding an element of $\mathcal{A}$, a binary functor $\mathcal{G}$ yielding an element of $\mathcal{A}$, a ternary functor $\mathcal{H}$ yielding an element of $\mathcal{A}$, and a binary functor $\mathcal{I}$ yielding an element of $\mathcal{A}$ and states that:

there exists a function F from WFF into $\mathcal{A}$ such that for every element p of WFF and for all elements d_1, d_2 of $\mathcal{A}$ holds if $p = \mathrm{VERUM}$, then $F(p) = \mathcal{B}$ but if p is atomic, then $F(p) = \mathcal{F}(p)$ but if p is negative and $d_1 = F(\mathrm{Arg}(p))$, then $F(p) = \mathcal{G}(d_1, p)$ but if p is conjunctive and $d_1 = F(\mathrm{LeftArg}(p))$ and $d_2 = F(\mathrm{RightArg}(p))$, then $F(p) = \mathcal{H}(d_1, d_2, p)$ but if p is universal and $d_1 = F(\mathrm{Scope}(p))$, then $F(p) = \mathcal{I}(d_1, p)$

for all values of the parameters.

The scheme *CQCF2_Func_Ex* deals with a non-empty set $\mathcal{A}$, a non-empty set $\mathcal{B}$, an element $\mathcal{C}$ of $\mathcal{B}^{\mathcal{A}}$, a ternary functor $\mathcal{F}$ yielding an element of $\mathcal{B}^{\mathcal{A}}$, a binary functor $\mathcal{G}$ yielding an element of $\mathcal{B}^{\mathcal{A}}$, a 4-ary functor $\mathcal{H}$ yielding an element of $\mathcal{B}^{\mathcal{A}}$, and a ternary functor $\mathcal{I}$ yielding an element of $\mathcal{B}^{\mathcal{A}}$ and states that:

there exists a function F from CQC-WFF into $\mathcal{B}^{\mathcal{A}}$ such that $F(\mathrm{VERUM}) = \mathcal{C}$ and for every k and for every variables list l of k and for every k-ary predicate symbol P holds $F(P[l]) = \mathcal{F}(k, P, l)$ and for all r, s, x and for all functions f,

g from $\mathcal{A}$ into $\mathcal{B}$ such that $f = F(r)$ and $g = F(s)$ holds $F(\neg r) = \mathcal{G}(f, r)$ and $F(r \wedge s) = \mathcal{H}(f, g, r, s)$ and $F(\forall_x r) = \mathcal{I}(x, f, r)$
for all values of the parameters.

The scheme *CQCF2_FUniq* concerns a non-empty set $\mathcal{A}$, a non-empty set $\mathcal{B}$, a function $\mathcal{C}$ from CQC-WFF into $\mathcal{B}^{\mathcal{A}}$, a function $\mathcal{D}$ from CQC-WFF into $\mathcal{B}^{\mathcal{A}}$, a function $\mathcal{E}$ from $\mathcal{A}$ into $\mathcal{B}$, a ternary functor $\mathcal{F}$ yielding a function from $\mathcal{A}$ into $\mathcal{B}$, a binary functor $\mathcal{G}$ yielding a function from $\mathcal{A}$ into $\mathcal{B}$, a 4-ary functor $\mathcal{H}$ yielding a function from $\mathcal{A}$ into $\mathcal{B}$, and a ternary functor $\mathcal{I}$ yielding a function from $\mathcal{A}$ into $\mathcal{B}$ and states that:

$\mathcal{C} = \mathcal{D}$

provided the parameters meet the following requirements:

- $\mathcal{C}(\mathrm{VERUM}) = \mathcal{E}$,
- for all k, l_1, P holds $\mathcal{C}(P[l_1]) = \mathcal{F}(k, P, l_1)$,
- Given r, s, x. Then for all functions f, g from $\mathcal{A}$ into $\mathcal{B}$ such that $f = \mathcal{C}(r)$ and $g = \mathcal{C}(s)$ holds $\mathcal{C}(\neg r) = \mathcal{G}(f, r)$ and $\mathcal{C}(r \wedge s) = \mathcal{H}(f, g, r, s)$ and $\mathcal{C}(\forall_x r) = \mathcal{I}(x, f, r)$,
- $\mathcal{D}(\mathrm{VERUM}) = \mathcal{E}$,
- for all k, l_1, P holds $\mathcal{D}(P[l_1]) = \mathcal{F}(k, P, l_1)$,
- Given r, s, x. Then for all functions f, g from $\mathcal{A}$ into $\mathcal{B}$ such that $f = \mathcal{D}(r)$ and $g = \mathcal{D}(s)$ holds $\mathcal{D}(\neg r) = \mathcal{G}(f, r)$ and $\mathcal{D}(r \wedge s) = \mathcal{H}(f, g, r, s)$ and $\mathcal{D}(\forall_x r) = \mathcal{I}(x, f, r)$.

We now state four propositions:

(10) p is a subformula of $\neg p$.

(11) p is a subformula of $p \wedge q$ and q is a subformula of $p \wedge q$.

(12) p is a subformula of $\forall_x p$.

(13) For every variables list l of k and for every i such that $1 \leq i$ and $i \leq \operatorname{len} l$ holds $l(i) \in \mathrm{BoundVar}$.

Let D be a non-empty set, and let f be a function from D into CQC-WFF. The functor $\mathrm{NEG}(f)$ yielding an element of CQC-WFF^D is defined as follows:

(Def.1) for every element a of D and for every element p of CQC-WFF such that $p = f(a)$ holds $(\mathrm{NEG}(f))(a) = \neg p$.

In the sequel f, h will denote elements of $\mathrm{BoundVar}^{\mathrm{BoundVar}}$ and K will denote a finite subset of BoundVar. Let f, g be functions from $[\!:\mathbb{N}, \mathrm{BoundVar}^{\mathrm{BoundVar}}:\!]$ into CQC-WFF, and let n be a natural number. The functor $\mathrm{CON}(f, g, n)$ yields an element of $\text{CQC-WFF}^{[\!:\mathbb{N}, \mathrm{BoundVar}^{\mathrm{BoundVar}}:\!]}$ and is defined by:

(Def.2) for all k, h, p, q such that $p = f(\langle k, h \rangle)$ and $q = g(\langle k + n, h \rangle)$ holds $(\mathrm{CON}(f, g, n))(\langle k, h \rangle) = p \wedge q$.

Let f be a function from $[\!:\mathbb{N}, \mathrm{BoundVar}^{\mathrm{BoundVar}}:\!]$ into CQC-WFF, and let x be a bound variable. The functor $\mathrm{UNIV}(x, f)$ yielding an element of $\text{CQC-WFF}^{[\!:\mathbb{N}, \mathrm{BoundVar}^{\mathrm{BoundVar}}:\!]}$ is defined by:

(Def.3) for all k, h, p such that $p = f(\langle k + 1, h +\cdot (\{x\} \longmapsto \mathrm{x}_k) \rangle)$ holds $(\mathrm{UNIV}(x, f))(\langle k, h \rangle) = \forall_{\mathrm{x}_k} p$.

Let us consider k, and let l be a variables list of k, and let f be an element of $\mathrm{BoundVar}^{\mathrm{BoundVar}}$. Then $f \cdot l$ is a variables list of k.

Let us consider k, and let P be a k-ary predicate symbol, and let l be a variables list of k. The functor $\mathrm{ATOM}(P, l)$ yields an element of

$\text{CQC-WFF}^{[\!:\mathbb{N},\, \mathrm{BoundVar}^{\mathrm{BoundVar}}:\!]}$

and is defined as follows:

(Def.4) for all n, h holds $(\mathrm{ATOM}(P, l))(\langle n, h\rangle) = P[h \cdot l]$.

Let us consider p. The number of quantifiers in p yields an element of $\mathbb{N}$ and is defined by the condition (Def.5).

(Def.5) There exists a function F from CQC-WFF into $\mathbb{N}$ such that the number of quantifiers in $p = F(p)$ and for all r, s, x, k and for every variables list l of k and for every k-ary predicate symbol P and for all elements r', s' of $\mathbb{N}$ such that $r' = F(r)$ and $s' = F(s)$ holds $F(\mathrm{VERUM}) = 0$ and $F(P[l]) = 0$ and $F(\neg r) = r'$ and $F(r \wedge s) = r' + s'$ and $F(\forall_x r) = r' + 1$.

Let f be a function from CQC-WFF into

$\text{CQC-WFF}^{[\!:\mathbb{N},\, \mathrm{BoundVar}^{\mathrm{BoundVar}}:\!]}$,

and let x be an element of CQC-WFF. Then $f(x)$ is an element of $\text{CQC-WFF}^{[\!:\mathbb{N},\, \mathrm{BoundVar}^{\mathrm{BoundVar}}:\!]}$.

The function Renum from CQC-WFF into $\text{CQC-WFF}^{[\!:\mathbb{N},\, \mathrm{BoundVar}^{\mathrm{BoundVar}}:\!]}$ is defined by the conditions (Def.6).

(Def.6) (i) $\mathrm{Renum}(\mathrm{VERUM}) = [\!:\mathbb{N},\, \mathrm{BoundVar}^{\mathrm{BoundVar}}:\!] \longmapsto \mathrm{VERUM}$,

(ii) for every k and for every variables list l of k and for every k-ary predicate symbol P holds $\mathrm{Renum}(P[l]) = \mathrm{ATOM}(P, l)$,

(iii) for all r, s, x and for all functions f, g from $[\!:\mathbb{N},\, \mathrm{BoundVar}^{\mathrm{BoundVar}}:\!]$ into CQC-WFF such that $f = \mathrm{Renum}(r)$ and $g = \mathrm{Renum}(s)$ holds $\mathrm{Renum}(\neg r) = \mathrm{NEG}(f)$ and $\mathrm{Renum}(r \wedge s) = \mathrm{CON}(f, g,$ the number of quantifiers in $r)$ and $\mathrm{Renum}(\forall_x r) = \mathrm{UNIV}(x, f)$.

Let us consider p, k, f. The functor $\mathrm{Renum}_{k,f}(p)$ yields an element of CQC-WFF and is defined by:

(Def.7) $\mathrm{Renum}_{k,f}(p) = \mathrm{Renum}(p)(\langle k, f\rangle)$.

Next we state several propositions:

(14) The number of quantifiers in VERUM $= 0$.

(15) The number of quantifiers in $P[l_1] = 0$.

(16) The number of quantifiers in $\neg p =$ the number of quantifiers in p.

(17) The number of quantifiers in $p \wedge q =$ (the number of quantifiers in p) + (the number of quantifiers in q).

(18) The number of quantifiers in $\forall_x p =$ (the number of quantifiers in p) + 1.

Let A be a non-empty subset of $\mathbb{N}$. The functor $\min A$ yields a natural number and is defined by:

(Def.8) $\min A \in A$ and for every k such that $k \in A$ holds $\min A \leq k$.

We now state two propositions:

(19) For all non-empty subsets A, B of $\mathbb{N}$ such that $A \subseteq B$ holds $\min B \leq \min A$.

(20) For every element p of WFF holds $\mathrm{snb}(p)$ is finite.

The scheme *MaxFinDomElem* concerns a non-empty set $\mathcal{A}$, a set $\mathcal{B}$, and a binary predicate $\mathcal{P}$, and states that:

there exists an element x of $\mathcal{A}$ such that $x \in \mathcal{B}$ and for every element y of $\mathcal{A}$ such that $y \in \mathcal{B}$ holds $\mathcal{P}[x, y]$

provided the parameters meet the following requirements:

- $\mathcal{B}$ is finite and $\mathcal{B} \neq \emptyset$ and $\mathcal{B} \subseteq \mathcal{A}$,
- for all elements x, y of $\mathcal{A}$ holds $\mathcal{P}[x, y]$ or $\mathcal{P}[y, x]$,
- for all elements x, y, z of $\mathcal{A}$ such that $\mathcal{P}[x, y]$ and $\mathcal{P}[y, z]$ holds $\mathcal{P}[x, z]$.

Let us consider p. The functor $\mathrm{NBI}(p)$ yielding a non-empty subset of $\mathbb{N}$ is defined as follows:

(Def.9) $\mathrm{NBI}(p) = \{k : \bigwedge_i [k \leq i \Rightarrow \mathrm{x}_i \notin \mathrm{snb}(p)]\}$.

Let us consider p. The functor $|\bullet : p|_{\mathbb{N}}$ yielding a natural number is defined as follows:

(Def.10) $|\bullet : p|_{\mathbb{N}} = \min \mathrm{NBI}(p)$.

Next we state several propositions:

(21) $|\bullet : p|_{\mathbb{N}} = 0$ if and only if p is closed.

(22) If $\mathrm{x}_i \in \mathrm{snb}(p)$, then $i < |\bullet : p|_{\mathbb{N}}$.

(23) $|\bullet : \mathrm{VERUM}|_{\mathbb{N}} = 0$.

(24) $|\bullet : \neg p|_{\mathbb{N}} = |\bullet : p|_{\mathbb{N}}$.

(25) $|\bullet : p|_{\mathbb{N}} \leq |\bullet : p \wedge q|_{\mathbb{N}}$ and $|\bullet : q|_{\mathbb{N}} \leq |\bullet : p \wedge q|_{\mathbb{N}}$.

Let C be a non-empty set, and let D be a non-empty subset of C. Then id_D is an element of D^D.

Let us consider p. The functor p with variables separated yielding an element of CQC-WFF is defined as follows:

(Def.11) p with variables separated $= \mathrm{Renum}_{|\bullet : p|_{\mathbb{N}}, \mathrm{id}_{\mathrm{BoundVar}}}(p)$.

The following proposition is true

(26) VERUM with variables separated = VERUM.

The scheme *CQCInd* deals with a unary predicate $\mathcal{P}$, and states that:

for every r holds $\mathcal{P}[r]$

provided the following requirements are met:

- $\mathcal{P}[\mathrm{VERUM}]$,
- for every k and for every variables list l of k and for every k-ary predicate symbol P holds $\mathcal{P}[P[l]]$,
- for every r such that $\mathcal{P}[r]$ holds $\mathcal{P}[\neg r]$,
- for all r, s such that $\mathcal{P}[r]$ and $\mathcal{P}[s]$ holds $\mathcal{P}[r \wedge s]$,
- for all r, x such that $\mathcal{P}[r]$ holds $\mathcal{P}[\forall_x r]$.

We now state four propositions:

(27) $P[l_1]$ with variables separated $= P[l_1]$.

(28) If p is atomic, then p with variables separated $= p$.

(29) $\neg p$ with variables separated $= \neg(p$ with variables separated$)$.

(30) If p is negative and $q = \mathrm{Arg}(p)$, then p with variables separated $= \neg(q$ with variables separated$)$.

Let us consider p, and let X be a subset of $[:\mathrm{CQC\text{-}WFF}, \mathbb{N}, \mathrm{Fin\,BoundVar}, \mathrm{BoundVar}^{\mathrm{BoundVar}}:]$. We say that X is closed w.r.t. p if and only if the conditions (Def.12) is satisfied.

(Def.12) (i) $\langle p, |\bullet : p|_{\mathbb{N}}, \emptyset_{\mathrm{BoundVar}}, \mathrm{id}_{\mathrm{BoundVar}}\rangle \in X$,

(ii) for all q, k, K, f such that $\langle \neg q, k, K, f\rangle \in X$ holds $\langle q, k, K, f\rangle \in X$,

(iii) for all q, r, k, K, f such that $\langle q \wedge r, k, K, f\rangle \in X$ holds $\langle q, k, K, f\rangle \in X$ and $\langle r, k +$ the number of quantifiers in $q, K, f\rangle \in X$,

(iv) for all q, x, k, K, f such that $\langle \forall_x q, k, K, f\rangle \in X$ holds $\langle q, k+1, K \cup \{x\}, f +\cdot (\{x\} \longmapsto \mathrm{x}_k)\rangle \in X$.

Let D be a non-empty set, and let x be an element of D. Then $\{x\}$ is an element of $\mathrm{Fin}\, D$.

Let us consider p. The functor $\mathbf{Quadruples}_p$ yields a subset of $[:\mathrm{CQC\text{-}WFF}, \mathbb{N}, \mathrm{Fin\,BoundVar}, \mathrm{BoundVar}^{\mathrm{BoundVar}}:]$ and is defined by:

(Def.13) $\mathbf{Quadruples}_p$ is closed w.r.t. p and for every subset D of $[:\mathrm{CQC\text{-}WFF}, \mathbb{N}, \mathrm{Fin\,BoundVar}, \mathrm{BoundVar}^{\mathrm{BoundVar}}:]$ such that D is closed w.r.t. p holds $\mathbf{Quadruples}_p \subseteq D$.

One can prove the following propositions:

(31) $\langle p, |\bullet : p|_{\mathbb{N}}, \emptyset_{\mathrm{BoundVar}}, \mathrm{id}_{\mathrm{BoundVar}}\rangle \in \mathbf{Quadruples}_p$.

(32) For all q, k, K, f such that $\langle \neg q, k, K, f\rangle \in \mathbf{Quadruples}_p$ holds $\langle q, k, K, f\rangle \in \mathbf{Quadruples}_p$.

(33) For all q, r, k, K, f such that $\langle q \wedge r, k, K, f\rangle \in \mathbf{Quadruples}_p$ holds $\langle q, k, K, f\rangle \in \mathbf{Quadruples}_p$ and $\langle r, k +$ the number of quantifiers in $q, K, f\rangle \in \mathbf{Quadruples}_p$.

(34) For all q, x, k, K, f such that $\langle \forall_x q, k, K, f\rangle \in \mathbf{Quadruples}_p$ holds $\langle q, k+1, K \cup \{x\}, f +\cdot (\{x\} \longmapsto \mathrm{x}_k)\rangle \in \mathbf{Quadruples}_p$.

(35) Suppose $\langle q, k, K, f\rangle \in \mathbf{Quadruples}_p$. Then

(i) $\langle q, k, K, f\rangle = \langle p, |\bullet : p|_{\mathbb{N}}, \emptyset_{\mathrm{BoundVar}}, \mathrm{id}_{\mathrm{BoundVar}}\rangle$, or

(ii) $\langle \neg q, k, K, f\rangle \in \mathbf{Quadruples}_p$, or

(iii) there exists r such that $\langle q \wedge r, k, K, f\rangle \in \mathbf{Quadruples}_p$, or

(iv) there exist r, l such that $k = l +$ the number of quantifiers in r and $\langle r \wedge q, l, K, f\rangle \in \mathbf{Quadruples}_p$, or

(v) there exist x, l, h such that $l + 1 = k$ and $h +\cdot (\{x\} \longmapsto \mathrm{x}_l) = f$ but $\langle \forall_x q, l, K, h\rangle \in \mathbf{Quadruples}_p$ or $\langle \forall_x q, l, K \setminus \{x\}, h\rangle \in \mathbf{Quadruples}_p$.

The scheme *Sep_regression* deals with an element $\mathcal{A}$ of CQC-WFF, and a 4-ary predicate $\mathcal{P}$, and states that:

for all q, k, K, f such that $\langle q, k, K, f\rangle \in \mathbf{Quadruples}_{\mathcal{A}}$ holds $\mathcal{P}[q, k, K, f]$ provided the following conditions are met:

- $\mathcal{P}[\mathcal{A}, |\bullet : \mathcal{A}|_{\mathbb{N}}, \emptyset_{\mathrm{BoundVar}}, \mathrm{id}_{\mathrm{BoundVar}}]$,
- for all q, k, K, f such that $\langle \neg q, k, K, f \rangle \in \mathbf{Quadruples}_{\mathcal{A}}$ and $\mathcal{P}[\neg q, k, K, f]$ holds $\mathcal{P}[q, k, K, f]$,
- for all q, r, k, K, f such that $\langle q \wedge r, k, K, f \rangle \in \mathbf{Quadruples}_{\mathcal{A}}$ and $\mathcal{P}[q \wedge r, k, K, f]$ holds $\mathcal{P}[q, k, K, f]$ and $\mathcal{P}[r, k +$ the number of quantifiers in $q, K, f]$,
- for all q, x, k, K, f such that $\langle \forall_x q, k, K, f \rangle \in \mathbf{Quadruples}_{\mathcal{A}}$ and $\mathcal{P}[\forall_x q, k, K, f]$ holds $\mathcal{P}[q, k+1, K \cup \{x\}, f +\!\cdot (\{x\} \longmapsto \mathrm{x}_k)]$.

We now state a number of propositions:

(36) For all q, k, K, f such that $\langle q, k, K, f \rangle \in \mathbf{Quadruples}_p$ holds q is a subformula of p.

(37) $\mathbf{Quadruples}_{\mathrm{VERUM}} = \{\langle \mathrm{VERUM}, 0, \emptyset_{\mathrm{BoundVar}}, \mathrm{id}_{\mathrm{BoundVar}} \rangle\}$.

(38) For every k and for every variables list l of k and for every k-ary predicate symbol P holds $\mathbf{Quadruples}_{P[l]} = \{\langle P[l], |\bullet : P[l]|_{\mathbb{N}}, \emptyset_{\mathrm{BoundVar}}, \mathrm{id}_{\mathrm{BoundVar}} \rangle\}$.

(39) For all q, k, K, f such that $\langle q, k, K, f \rangle \in \mathbf{Quadruples}_p$ holds $\mathrm{snb}(q) \subseteq \mathrm{snb}(p) \cup K$.

(40) If $\langle q, m, K, f \rangle \in \mathbf{Quadruples}_p$ and $\mathrm{x}_i \in f\,^\circ\, K$, then $i < m$.

(41) If $\langle q, m, K, f \rangle \in \mathbf{Quadruples}_p$, then $\mathrm{x}_m \notin f\,^\circ\, K$.

(42) If $\langle q, m, K, f \rangle \in \mathbf{Quadruples}_p$ and $\mathrm{x}_i \in f\,^\circ\, \mathrm{snb}(p)$, then $i < m$.

(43) If $\langle q, m, K, f \rangle \in \mathbf{Quadruples}_p$ and $\mathrm{x}_i \in f\,^\circ\, \mathrm{snb}(q)$, then $i < m$.

(44) If $\langle q, m, K, f \rangle \in \mathbf{Quadruples}_p$, then $\mathrm{x}_m \notin f\,^\circ\, \mathrm{snb}(q)$.

(45) $\mathrm{snb}(p) = \mathrm{snb}(p$ with variables separated$)$.

(46) $|\bullet : p|_{\mathbb{N}} = |\bullet : p$ with variables separated$|_{\mathbb{N}}$.

Let us consider p, q. We say that p and q are similar if and only if:

(Def.14) p with variables separated $= q$ with variables separated.

One can prove the following propositions:

(47) p and p are similar.

(48) If p and q are similar, then q and p are similar.

(49) If p and q are similar and q and r are similar, then p and r are similar.

References

[1] Grzegorz Bancerek. Connectives and subformulae of the first order language. *Formalized Mathematics*, 1(**3**):451–458, 1990.

[2] Grzegorz Bancerek. The fundamental properties of natural numbers. *Formalized Mathematics*, 1(**1**):41–46, 1990.

[3] Grzegorz Bancerek and Krzysztof Hryniewiecki. Segments of natural numbers and finite sequences. *Formalized Mathematics*, 1(**1**):107–114, 1990.

[4] Czesław Byliński. A classical first order language. *Formalized Mathematics*, 1(**4**):669–676, 1990.

[5] Czesław Byliński. Functions and their basic properties. *Formalized Mathematics*, 1(**1**):55–65, 1990.

[6] Czesław Byliński. Functions from a set to a set. *Formalized Mathematics*, 1(**1**):153–164, 1990.

[7] Czesław Byliński. The modification of a function by a function and the iteration of the composition of a function. *Formalized Mathematics*, 1(**3**):521–527, 1990.
[8] Czesław Byliński. Partial functions. *Formalized Mathematics*, 1(**2**):357–367, 1990.
[9] Czesław Byliński and Grzegorz Bancerek. Variables in formulae of the first order language. *Formalized Mathematics*, 1(**3**):459–469, 1990.
[10] Agata Darmochwał. Finite sets. *Formalized Mathematics*, 1(**1**):165–167, 1990.
[11] Krzysztof Hryniewiecki. Basic properties of real numbers. *Formalized Mathematics*, 1(**1**):35–40, 1990.
[12] Włodzimierz Lesisz and Witold A. Pogorzelski. A simplified definition of the notion of similarity between formulas of the first order predicate calculus. *Reports on Mathematical Logic*, (7):63–69, 1976.
[13] Beata Padlewska. Families of sets. *Formalized Mathematics*, 1(**1**):147–152, 1990.
[14] Witold A. Pogorzelski. *Klasyczny Rachunek Predykatów*. PWN, Warszawa, 1981.
[15] Witold A. Pogorzelski and Tadeusz Prucnal. The substitution rule for predicate letters in the first-order predicate calculus. *Reports on Mathematical Logic*, (5):77–90, 1975.
[16] Helena Rasiowa and Roman Sikorski. *The Mathematics of Metamathematics*. Volume 41 of *Monografie Matematyczne*, PWN, Warszawa, 1968.
[17] Piotr Rudnicki and Andrzej Trybulec. A first order language. *Formalized Mathematics*, 1(**2**):303–311, 1990.
[18] Andrzej Trybulec. Binary operations applied to functions. *Formalized Mathematics*, 1(**2**):329–334, 1990.
[19] Andrzej Trybulec. Domains and their Cartesian products. *Formalized Mathematics*, 1(**1**):115–122, 1990.
[20] Andrzej Trybulec. Enumerated sets. *Formalized Mathematics*, 1(**1**):25–34, 1990.
[21] Andrzej Trybulec. Function domains and Frænkel operator. *Formalized Mathematics*, 1(**3**):495–500, 1990.
[22] Andrzej Trybulec. Semilattice operations on finite subsets. *Formalized Mathematics*, 1(**2**):369–376, 1990.
[23] Andrzej Trybulec. Tarski Grothendieck set theory. *Formalized Mathematics*, 1(**1**):9–11, 1990.
[24] Andrzej Trybulec. Tuples, projections and Cartesian products. *Formalized Mathematics*, 1(**1**):97–105, 1990.
[25] Andrzej Trybulec and Czesław Byliński. Some properties of real numbers. *Formalized Mathematics*, 1(**3**):445–449, 1990.
[26] Andrzej Trybulec and Agata Darmochwał. Boolean domains. *Formalized Mathematics*, 1(**1**):187–190, 1990.
[27] Zinaida Trybulec. Properties of subsets. *Formalized Mathematics*, 1(**1**):67–71, 1990.

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