FORMALIZED MATHEMATICS
Vol.2, No.3, May–August 1991
Université Catholique de Louvain

Shear Theorems and Their Role in Affine Geometry

Jolanta Świerzyńska
Warsaw University
Białystok

Bogdan Świerzyński
Warsaw University
Białystok

Summary. Investigations on affine shear theorems, major and minor, direct and indirect. We prove logical relationships which hold between these statements and between them and other classical affine configurational axioms (eg. minor and major Pappus Axiom, Desargues Axioms et al.). For the shear, Desargues and Pappus Axioms formulated in terms of metric affine spaces we prove that they are equivalent to corresponding statements formulated in terms of affine reduct of the given space.

MML Identifier: CONMETR1.

The terminology and notation used in this paper have been introduced in the following papers: [2], [4], [1], [3], [6], [7], and [5]. We follow a convention: X will be an affine plane, o, a_1, a_2, a_3, a_4, b_1, b_2, b_3, b_4 will be elements of the points of X, and M, N will be subsets of the points of X. Let us consider X. We say that X satisfies minor Scherungssatz if and only if the condition (Def.1) is satisfied.

(Def.1) Given a_1, a_2, a_3, a_4, b_1, b_2, b_3, b_4, M, N. Suppose that
- (i) $M \parallel N$,
- (ii) $a_1 \in M$,
- (iii) $a_3 \in M$,
- (iv) $b_1 \in M$,
- (v) $b_3 \in M$,
- (vi) $a_2 \in N$,
- (vii) $a_4 \in N$,
- (viii) $b_2 \in N$,
- (ix) $b_4 \in N$,
- (x) $a_4 \notin M$,

ISSN 0777–4028

(xi) $a_2 \notin M$,
(xii) $b_2 \notin M$,
(xiii) $b_4 \notin M$,
(xiv) $a_1 \notin N$,
(xv) $a_3 \notin N$,
(xvi) $b_1 \notin N$,
(xvii) $b_3 \notin N$,
(xviii) $a_3, a_2 \parallel b_3, b_2$,
(xix) $a_2, a_1 \parallel b_2, b_1$,
(xx) $a_1, a_4 \parallel b_1, b_4$.

Then $a_3, a_4 \parallel b_3, b_4$.

Let us consider X. We say that X satisfies major Scherungssatz if and only if the condition (Def.2) is satisfied.

(Def.2) Given o, a_1, a_2, a_3, a_4, b_1, b_2, b_3, b_4, M, N. Suppose that
(i) M is a line,
(ii) N is a line,
(iii) $o \in M$,
(iv) $o \in N$,
(v) $a_1 \in M$,
(vi) $a_3 \in M$,
(vii) $b_1 \in M$,
(viii) $b_3 \in M$,
(ix) $a_2 \in N$,
(x) $a_4 \in N$,
(xi) $b_2 \in N$,
(xii) $b_4 \in N$,
(xiii) $a_4 \notin M$,
(xiv) $a_2 \notin M$,
(xv) $b_2 \notin M$,
(xvi) $b_4 \notin M$,
(xvii) $a_1 \notin N$,
(xviii) $a_3 \notin N$,
(xix) $b_1 \notin N$,
(xx) $b_3 \notin N$,
(xxi) $a_3, a_2 \parallel b_3, b_2$,
(xxii) $a_2, a_1 \parallel b_2, b_1$,
(xxiii) $a_1, a_4 \parallel b_1, b_4$.

Then $a_3, a_4 \parallel b_3, b_4$.

Let us consider X. We say that X satisfies Scherungssatz if and only if the condition (Def.3) is satisfied.

(Def.3) Given a_1, a_2, a_3, a_4, b_1, b_2, b_3, b_4, M, N. Suppose that
(i) M is a line,
(ii) N is a line,
(iii) $a_1 \in M$,

(iv) $a_3 \in M$,
(v) $b_1 \in M$,
(vi) $b_3 \in M$,
(vii) $a_2 \in N$,
(viii) $a_4 \in N$,
(ix) $b_2 \in N$,
(x) $b_4 \in N$,
(xi) $a_4 \notin M$,
(xii) $a_2 \notin M$,
(xiii) $b_2 \notin M$,
(xiv) $b_4 \notin M$,
(xv) $a_1 \notin N$,
(xvi) $a_3 \notin N$,
(xvii) $b_1 \notin N$,
(xviii) $b_3 \notin N$,
(xix) $a_3, a_2 \parallel b_3, b_2$,
(xx) $a_2, a_1 \parallel b_2, b_1$,
(xxi) $a_1, a_4 \parallel b_1, b_4$.

Then $a_3, a_4 \parallel b_3, b_4$.

Let us consider X. We say that X satisfies Scherungssatz* if and only if the condition (Def.4) is satisfied.

(Def.4) Given a_1, a_2, a_3, a_4, b_1, b_2, b_3, b_4, M, N. Suppose that
(i) M is a line,
(ii) N is a line,
(iii) $a_1 \in M$,
(iv) $a_3 \in M$,
(v) $b_2 \in M$,
(vi) $b_4 \in M$,
(vii) $a_2 \in N$,
(viii) $a_4 \in N$,
(ix) $b_1 \in N$,
(x) $b_3 \in N$,
(xi) $a_4 \notin M$,
(xii) $a_2 \notin M$,
(xiii) $b_1 \notin M$,
(xiv) $b_3 \notin M$,
(xv) $a_1 \notin N$,
(xvi) $a_3 \notin N$,
(xvii) $b_2 \notin N$,
(xviii) $b_4 \notin N$,
(xix) $a_3, a_2 \parallel b_3, b_2$,
(xx) $a_2, a_1 \parallel b_2, b_1$,
(xxi) $a_1, a_4 \parallel b_1, b_4$.

Then $a_3, a_4 \parallel b_3, b_4$.

Let us consider X. We say that X satisfies minor Scherungssatz* if and only if the condition (Def.5) is satisfied.

(Def.5) Given a_1, a_2, a_3, a_4, b_1, b_2, b_3, b_4, M, N. Suppose that

(i) $M \parallel N$,
(ii) $a_1 \in M$,
(iii) $a_3 \in M$,
(iv) $b_2 \in M$,
(v) $b_4 \in M$,
(vi) $a_2 \in N$,
(vii) $a_4 \in N$,
(viii) $b_1 \in N$,
(ix) $b_3 \in N$,
(x) $a_4 \notin M$,
(xi) $a_2 \notin M$,
(xii) $b_1 \notin M$,
(xiii) $b_3 \notin M$,
(xiv) $a_1 \notin N$,
(xv) $a_3 \notin N$,
(xvi) $b_2 \notin N$,
(xvii) $b_4 \notin N$,
(xviii) $a_3, a_2 \parallel b_3, b_2$,
(xix) $a_2, a_1 \parallel b_2, b_1$,
(xx) $a_1, a_4 \parallel b_1, b_4$.

Then $a_3, a_4 \parallel b_3, b_4$.

Let us consider X. We say that X satisfies major Scherungssatz* if and only if the condition (Def.6) is satisfied.

(Def.6) Given o, a_1, a_2, a_3, a_4, b_1, b_2, b_3, b_4, M, N. Suppose that

(i) M is a line,
(ii) N is a line,
(iii) $o \in M$,
(iv) $o \in N$,
(v) $a_1 \in M$,
(vi) $a_3 \in M$,
(vii) $b_2 \in M$,
(viii) $b_4 \in M$,
(ix) $a_2 \in N$,
(x) $a_4 \in N$,
(xi) $b_1 \in N$,
(xii) $b_3 \in N$,
(xiii) $a_4 \notin M$,
(xiv) $a_2 \notin M$,
(xv) $b_1 \notin M$,
(xvi) $b_3 \notin M$,
(xvii) $a_1 \notin N$,

(xviii) $a_3 \notin N$,
(xix) $b_2 \notin N$,
(xx) $b_4 \notin N$,
(xxi) $a_3, a_2 \parallel b_3, b_2$,
(xxii) $a_2, a_1 \parallel b_2, b_1$,
(xxiii) $a_1, a_4 \parallel b_1, b_4$.
Then $a_3, a_4 \parallel b_3, b_4$.

Next we state a number of propositions:

(1) X satisfies Scherungssatz* if and only if X satisfies minor Scherungssatz* and X satisfies major Scherungssatz*.

(2) X satisfies Scherungssatz if and only if X satisfies minor Scherungssatz and X satisfies major Scherungssatz.

(3) If X satisfies minor Scherungssatz*, then X satisfies minor Scherungssatz.

(4) If X satisfies major Scherungssatz*, then X satisfies major Scherungssatz.

(5) If X satisfies Scherungssatz*, then X satisfies Scherungssatz.

(6) If X satisfies **des**, then X satisfies minor Scherungssatz.

(7) If X satisfies **DES**, then X satisfies major Scherungssatz.

(8) X satisfies **DES** if and only if X satisfies Scherungssatz.

(9) X satisfies **pap** if and only if X satisfies minor Scherungssatz*.

(10) X satisfies **PAP** if and only if X satisfies major Scherungssatz*.

(11) X satisfies **PPAP** if and only if X satisfies Scherungssatz*.

(12) If X satisfies major Scherungssatz*, then X satisfies minor Scherungssatz *.

In the sequel X denotes a metric affine plane. We now state several propositions:

(13) The affine reduct of X satisfies Scherungssatz if and only if Scherungssatz holds in X.

(14) trapezium variant of Desargues Axiom holds in X if and only if the affine reduct of X satisfies **TDES**.

(15) The affine reduct of X satisfies **des** if and only if minor Desargues Axiom holds in X.

(16) Pappos Axiom holds in X if and only if the affine reduct of X satisfies **PAP**.

(17) Desargues Axiom holds in X if and only if the affine reduct of X satisfies **DES**.

References

[1] Henryk Oryszczyszyn and Krzysztof Prażmowski. Analytical metric affine spaces and planes. *Formalized Mathematics*, 1(**5**):891–899, 1990.

[2] Henryk Oryszczyszyn and Krzysztof Prażmowski. Analytical ordered affine spaces. *Formalized Mathematics*, 1(**3**):601–605, 1990.

[3] Henryk Oryszczyszyn and Krzysztof Prażmowski. Classical configurations in affine planes. *Formalized Mathematics*, 1(**4**):625–633, 1990.

[4] Henryk Oryszczyszyn and Krzysztof Prażmowski. Ordered affine spaces defined in terms of directed parallelity - part I. *Formalized Mathematics*, 1(**3**):611–615, 1990.

[5] Henryk Oryszczyszyn and Krzysztof Prażmowski. Translations in affine planes. *Formalized Mathematics*, 1(**4**):751–753, 1990.

[6] Jolanta Świerzyńska and Bogdan Świerzyński. Metric-affine configurations in metric affine planes - Part I. *Formalized Mathematics*, 2(**3**):331–334, 1991.

[7] Jolanta Świerzyńska and Bogdan Świerzyński. Metric-affine configurations in metric affine planes - Part II. *Formalized Mathematics*, 2(**3**):335–340, 1991.

Received April 19, 1991