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Metric-Affine Configurations in Metric Affine Planes - Part II

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Summary. A continuation of [5]. We introduce more configurational axioms i.e. orthogonalizations of "scherungssatzes" (direct and indirect), "Scherungssatz" with orthogonal axes, Pappus axiom with orthogonal axes; we also consider the affine Major Pappus Axiom and affine minor Desargues Axiom. We prove a number of implications which hold between the above axioms.

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The articles [2], [4], [1], [3], and [5] provide the notation and terminology for this paper. We adopt the following rules: X will denote a metric affine plane, o, a, a_1, a_2, a_3, a_4, b, b_1, b_2, b_3, b_4, c, c_1, d will denote elements of the points of X, and A, K, M, N will denote subsets of the points of X. Let us consider X. We say that Pappos Axiom with orthogonal axes holds in X if and only if the condition (Def.1) is satisfied.

(Def.1) Given o, a_1, a_2, a_3, b_1, b_2, b_3, M, N. Suppose that
- (i) $o \in M$,
- (ii) $a_1 \in M$,
- (iii) $a_2 \in M$,
- (iv) $a_3 \in M$,
- (v) $o \in N$,
- (vi) $b_1 \in N$,
- (vii) $b_2 \in N$,
- (viii) $b_3 \in N$,
- (ix) $b_2 \notin M$,
- (x) $a_3 \notin N$,
- (xi) $M \perp N$,
- (xii) $o \neq a_1$,

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(xiii) $o \neq a_2$,
(xiv) $o \neq a_3$,
(xv) $o \neq b_1$,
(xvi) $o \neq b_2$,
(xvii) $o \neq b_3$,
(xviii) $a_3, b_2 \parallel a_2, b_1$,
(xix) $a_3, b_3 \parallel a_1, b_1$.
Then $a_1, b_2 \parallel a_2, b_3$.

Let us consider X. We say that Pappos Axiom holds in X if and only if the condition (Def.2) is satisfied.

(Def.2) Given o, a_1, a_2, a_3, b_1, b_2, b_3, M, N. Suppose that
(i) M is a line,
(ii) N is a line,
(iii) $o \in M$,
(iv) $a_1 \in M$,
(v) $a_2 \in M$,
(vi) $a_3 \in M$,
(vii) $o \in N$,
(viii) $b_1 \in N$,
(ix) $b_2 \in N$,
(x) $b_3 \in N$,
(xi) $b_2 \notin M$,
(xii) $a_3 \notin N$,
(xiii) $o \neq a_1$,
(xiv) $o \neq a_2$,
(xv) $o \neq a_3$,
(xvi) $o \neq b_1$,
(xvii) $o \neq b_2$,
(xviii) $o \neq b_3$,
(xix) $a_3, b_2 \parallel a_2, b_1$,
(xx) $a_3, b_3 \parallel a_1, b_1$.
Then $a_1, b_2 \parallel a_2, b_3$.

Let us consider X. We say that MH1 holds in X if and only if the condition (Def.3) is satisfied.

(Def.3) Given a_1, a_2, a_3, a_4, b_1, b_2, b_3, b_4, M, N. Suppose that
(i) $M \perp N$,
(ii) $a_1 \in M$,
(iii) $a_3 \in M$,
(iv) $b_1 \in M$,
(v) $b_3 \in M$,
(vi) $a_2 \in N$,
(vii) $a_4 \in N$,
(viii) $b_2 \in N$,
(ix) $b_4 \in N$,

(x) $a_2 \notin M$,
(xi) $a_4 \notin M$,
(xii) $a_1, a_2 \perp b_1, b_2$,
(xiii) $a_2, a_3 \perp b_2, b_3$,
(xiv) $a_3, a_4 \perp b_3, b_4$.
Then $a_1, a_4 \perp b_1, b_4$.

Let us consider X. We say that MH2 holds in X if and only if the condition (Def.4) is satisfied.

(Def.4) Given a_1, a_2, a_3, a_4, b_1, b_2, b_3, b_4, M, N. Suppose that
(i) $M \perp N$,
(ii) $a_1 \in M$,
(iii) $a_3 \in M$,
(iv) $b_2 \in M$,
(v) $b_4 \in M$,
(vi) $a_2 \in N$,
(vii) $a_4 \in N$,
(viii) $b_1 \in N$,
(ix) $b_3 \in N$,
(x) $a_2 \notin M$,
(xi) $a_4 \notin M$,
(xii) $a_1, a_2 \perp b_1, b_2$,
(xiii) $a_2, a_3 \perp b_2, b_3$,
(xiv) $a_3, a_4 \perp b_3, b_4$.
Then $a_1, a_4 \perp b_1, b_4$.

Let us consider X. We say that trapezium variant of Desargues Axiom holds in X if and only if the condition (Def.5) is satisfied.

(Def.5) Given o, a, a_1, b, b_1, c, c_1. Suppose that
(i) $o \neq a$,
(ii) $o \neq a_1$,
(iii) $o \neq b$,
(iv) $o \neq b_1$,
(v) $o \neq c$,
(vi) $o \neq c_1$,
(vii) not $\mathbf{L}(b, b_1, a)$,
(viii) not $\mathbf{L}(b, b_1, c)$,
(ix) $\mathbf{L}(o, a, a_1)$,
(x) $\mathbf{L}(o, b, b_1)$,
(xi) $\mathbf{L}(o, c, c_1)$,
(xii) $a, b \parallel a_1, b_1$,
(xiii) $a, b \parallel o, c$,
(xiv) $b, c \parallel b_1, c_1$.
Then $a, c \parallel a_1, c_1$.

Let us consider X. We say that Scherungssatz holds in X if and only if the condition (Def.6) is satisfied.

(Def.6) Given a_1, a_2, a_3, a_4, b_1, b_2, b_3, b_4, M, N. Suppose that

(i) M is a line,
(ii) N is a line,
(iii) $a_1 \in M$,
(iv) $a_3 \in M$,
(v) $b_1 \in M$,
(vi) $b_3 \in M$,
(vii) $a_2 \in N$,
(viii) $a_4 \in N$,
(ix) $b_2 \in N$,
(x) $b_4 \in N$,
(xi) $a_4 \notin M$,
(xii) $a_2 \notin M$,
(xiii) $b_2 \notin M$,
(xiv) $b_4 \notin M$,
(xv) $a_1 \notin N$,
(xvi) $a_3 \notin N$,
(xvii) $b_1 \notin N$,
(xviii) $b_3 \notin N$,
(xix) $a_3, a_2 \parallel b_3, b_2$,
(xx) $a_2, a_1 \parallel b_2, b_1$,
(xxi) $a_1, a_4 \parallel b_1, b_4$.

Then $a_3, a_4 \parallel b_3, b_4$.

Let us consider X. We say that Scherungssatz with orthogonal axes holds in X if and only if the condition (Def.7) is satisfied.

(Def.7) Given a_1, a_2, a_3, a_4, b_1, b_2, b_3, b_4, M, N. Suppose that

(i) $M \perp N$,
(ii) $a_1 \in M$,
(iii) $a_3 \in M$,
(iv) $b_1 \in M$,
(v) $b_3 \in M$,
(vi) $a_2 \in N$,
(vii) $a_4 \in N$,
(viii) $b_2 \in N$,
(ix) $b_4 \in N$,
(x) $a_4 \notin M$,
(xi) $a_2 \notin M$,
(xii) $b_2 \notin M$,
(xiii) $b_4 \notin M$,
(xiv) $a_1 \notin N$,
(xv) $a_3 \notin N$,
(xvi) $b_1 \notin N$,

(xvii) $b_3 \notin N$,
(xviii) $a_3, a_2 \parallel b_3, b_2$,
(xix) $a_2, a_1 \parallel b_2, b_1$,
(xx) $a_1, a_4 \parallel b_1, b_4$.
Then $a_3, a_4 \parallel b_3, b_4$.

Let us consider X. We say that minor Desargues Axiom holds in X if and only if:

(Def.8) for all a, a_1, b, b_1, c, c_1 such that not $\mathbf{L}(a, a_1, b)$ and not $\mathbf{L}(a, a_1, c)$ and $a, a_1 \parallel b, b_1$ and $a, a_1 \parallel c, c_1$ and $a, b \parallel a_1, b_1$ and $a, c \parallel a_1, c_1$ holds $b, c \parallel b_1, c_1$.

One can prove the following propositions:

(1) There exist a, b, c such that $\mathbf{L}(a, b, c)$ and $a \neq b$ and $b \neq c$ and $c \neq a$.

(2) For all a, b such that $a \neq b$ there exists c such that $\mathbf{L}(a, b, c)$ and $a \neq c$ and $b \neq c$.

(3) For all A, a such that A is a line there exists K such that $a \in K$ and $A \perp K$.

(4) If A is a line and $a \in A$ and $b \in A$ and $c \in A$, then $\mathbf{L}(a, b, c)$.

(5) If A is a line and M is a line and $a \in A$ and $b \in A$ and $a \in M$ and $b \in M$, then $a = b$ or $A = M$.

(6) For all a, b, c, d, M and for every subset M' of the points of the affine reduct of X and for all elements c', d' of the points of the affine reduct of X such that $c = c'$ and $d = d'$ and $M = M'$ and $a \in M$ and $b \in M$ and $c', d' \parallel M'$ holds $c, d \parallel a, b$.

(7) If trapezium variant of Desargues Axiom holds in X, then the affine reduct of X satisfies **TDES**.

(8) If the affine reduct of X satisfies **des**, then minor Desargues Axiom holds in X.

(9) If MH1 holds in X, then Scherungssatz with orthogonal axes holds in X.

(10) If MH2 holds in X, then Scherungssatz with orthogonal axes holds in X.

(11) If AH holds in X, then trapezium variant of Desargues Axiom holds in X.

(12) If Scherungssatz with orthogonal axes holds in X and trapezium variant of Desargues Axiom holds in X, then Scherungssatz holds in X.

(13) If Pappos Axiom with orthogonal axes holds in X and Desargues Axiom holds in X, then Pappos Axiom holds in X.

(14) If MH1 holds in X and MH2 holds in X, then Pappos Axiom with orthogonal axes holds in X.

(15) If theorem on three perpendiculars holds in X, then Pappos Axiom with orthogonal axes holds in X.

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