# Calculus of Propositions [1]

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**Summary.** Continues the analysis of the classical language of first order (see [6], [1], [3], [4], [2]). Three connectives : truth, negation and conjuction are primary (see [6]). The others (alternative, implication and equivalence) are defined with respect to them (see [1]). We prove some important tautologies of the calculus of propositions. Most of them are given as axioms of the classical logical calculus (see [5]). In the last part of our article we give some basic rules of inference.

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The notation and terminology used here have been introduced in the papers [3] and [4]. In the sequel $p$, $q$, $r$, $s$ are elements of CQC$-$WFF. One can prove the following propositions:

(1)   $\neg(p \wedge \neg p) \in \text{Taut}$.

(2)   $p \vee \neg p \in \text{Taut}$.

(3)   $p \Rightarrow p \vee q \in \text{Taut}$.

(4)   $q \Rightarrow p \vee q \in \text{Taut}$.

(5)   $p \vee q \Rightarrow (\neg p \Rightarrow q) \in \text{Taut}$.

(6)   $\neg(p \vee q) \Rightarrow \neg p \wedge \neg q \in \text{Taut}$.

(7)   $\neg p \wedge \neg q \Rightarrow \neg(p \vee q) \in \text{Taut}$.

(8)   $p \vee q \Rightarrow q \vee p \in \text{Taut}$.

(9)   $\neg p \vee p \in \text{Taut}$.

(10)   $\neg(p \vee q) \Rightarrow \neg p \in \text{Taut}$.

(11)   $p \vee p \Rightarrow p \in \text{Taut}$.

(12)   $p \Rightarrow p \vee p \in \text{Taut}$.

(13)   $p \wedge \neg p \Rightarrow q \in \text{Taut}$.

(14)   $(p \Rightarrow q) \Rightarrow \neg p \vee q \in \text{Taut}$.

(15)     $p \wedge q \Rightarrow \neg(p \Rightarrow \neg q) \in$ Taut.

(16)     $\neg(p \Rightarrow \neg q) \Rightarrow p \wedge q \in$ Taut.

(17)     $\neg(p \wedge q) \Rightarrow \neg p \vee \neg q \in$ Taut.

(18)     $\neg p \vee \neg q \Rightarrow \neg(p \wedge q) \in$ Taut.

(19)     $p \wedge q \Rightarrow p \in$ Taut.

(20)     $p \wedge q \Rightarrow p \vee q \in$ Taut.

(21)     $p \wedge q \Rightarrow q \in$ Taut.

(22)     $p \Rightarrow p \wedge p \in$ Taut.

(23)     $(p \Leftrightarrow q) \Rightarrow (p \Rightarrow q) \in$ Taut.

(24)     $(p \Leftrightarrow q) \Rightarrow (q \Rightarrow p) \in$ Taut.

(25)     $p \vee q \vee r \Rightarrow p \vee (q \vee r) \in$ Taut.

(26)     $p \wedge q \wedge r \Rightarrow p \wedge (q \wedge r) \in$ Taut.

(27)     $p \vee (q \vee r) \Rightarrow p \vee q \vee r \in$ Taut.

(28)     $p \Rightarrow (q \Rightarrow p \wedge q) \in$ Taut.

(29)     $(p \Rightarrow q) \Rightarrow ((q \Rightarrow p) \Rightarrow (p \Leftrightarrow q)) \in$ Taut.

(30)     $p \vee q \Leftrightarrow q \vee p \in$ Taut.

(31)     $(p \wedge q \Rightarrow r) \Rightarrow (p \Rightarrow (q \Rightarrow r)) \in$ Taut.

The following propositions are true:

(32)     $(p \Rightarrow (q \Rightarrow r)) \Rightarrow (p \wedge q \Rightarrow r) \in$ Taut.

(33)     $(r \Rightarrow p) \Rightarrow ((r \Rightarrow q) \Rightarrow (r \Rightarrow p \wedge q)) \in$ Taut.

(34)     $(p \vee q \Rightarrow r) \Rightarrow (p \Rightarrow r) \vee (q \Rightarrow r) \in$ Taut.

(35)     $(p \Rightarrow r) \Rightarrow ((q \Rightarrow r) \Rightarrow (p \vee q \Rightarrow r)) \in$ Taut.

(36)     $(p \Rightarrow r) \wedge (q \Rightarrow r) \Rightarrow (p \vee q \Rightarrow r) \in$ Taut.

(37)     $(p \Rightarrow q \wedge \neg q) \Rightarrow \neg p \in$ Taut.

(38)     $(p \vee q) \wedge (p \vee r) \Rightarrow p \vee q \wedge r \in$ Taut.

(39)     $p \wedge (q \vee r) \Rightarrow p \wedge q \vee p \wedge r \in$ Taut.

(40)     $(p \vee r) \wedge (q \vee r) \Rightarrow p \wedge q \vee r \in$ Taut.

(41)     $(p \vee q) \wedge r \Rightarrow p \wedge r \vee q \wedge r \in$ Taut.

(42)     If $p \in$ Taut, then $p \vee q \in$ Taut.

(43)     If $q \in$ Taut, then $p \vee q \in$ Taut.

(44)     If $p \wedge q \in$ Taut, then $p \in$ Taut.

(45)     If $p \wedge q \in$ Taut, then $q \in$ Taut.

(46)     If $p \wedge q \in$ Taut, then $p \vee q \in$ Taut.

(47)     If $p \in$ Taut and $q \in$ Taut, then $p \wedge q \in$ Taut.

(48)     If $p \Rightarrow q \in$ Taut, then $p \vee r \Rightarrow q \vee r \in$ Taut.

(49)     If $p \Rightarrow q \in$ Taut, then $r \vee p \Rightarrow r \vee q \in$ Taut.

(50)     If $p \Rightarrow q \in$ Taut, then $r \wedge p \Rightarrow r \wedge q \in$ Taut.

(51)     If $p \Rightarrow q \in$ Taut, then $p \wedge r \Rightarrow q \wedge r \in$ Taut.

(52)     If $r \Rightarrow p \in$ Taut and $r \Rightarrow q \in$ Taut, then $r \Rightarrow p \wedge q \in$ Taut.

(53)　　If $p \Rightarrow r \in$ Taut and $q \Rightarrow r \in$ Taut, then $p \vee q \Rightarrow r \in$ Taut.

(54)　　If $p \vee q \in$ Taut and $\neg p \in$ Taut, then $q \in$ Taut.

(55)　　If $p \vee q \in$ Taut and $\neg q \in$ Taut, then $p \in$ Taut.

(56)　　If $p \Rightarrow q \in$ Taut and $r \Rightarrow s \in$ Taut, then $p \wedge r \Rightarrow q \wedge s \in$ Taut.

(57)　　If $p \Rightarrow q \in$ Taut and $r \Rightarrow s \in$ Taut, then $p \vee r \Rightarrow q \vee s \in$ Taut.

(58)　　If $p \wedge \neg q \Rightarrow \neg p \in$ Taut, then $p \Rightarrow q \in$ Taut.

# References

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