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Subsets of Complex Numbers

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The articles [7], [6], [9], [10], [4], [5], [1], [2], [3], and [8] provide the notation and terminology for this paper.

1. Main Part

The functor $\mathbb{R}$ is defined by:

(Def. 1) $\mathbb{R} = (\mathbb{R}_+ \cup [: \{0\}, \mathbb{R}_+ :]) \setminus \{\langle 0, 0\rangle\}$.

We introduce $\mathbb{N}$ as a synonym of ω.

Let us observe that $\mathbb{R}$ is non empty.

The functor $\mathbb{C}$ is defined as follows:

(Def. 2) $\mathbb{C} = (\mathbb{R}^{\{0,\mathbf{1}\}} \setminus \{x; x$ ranges over elements of $\mathbb{R}^{\{0,\mathbf{1}\}}$: $x(\mathbf{1}) = 0\}) \cup \mathbb{R}$.

The functor $\mathbb{Q}$ is defined by:

(Def. 3) $\mathbb{Q} = (\mathbb{Q}_+ \cup [: \{0\}, \mathbb{Q}_+ :]) \setminus \{\langle 0, 0\rangle\}$.

The functor $\mathbb{Z}$ is defined as follows:

(Def. 4) $\mathbb{Z} = (\mathbb{N} \cup [: \{0\}, \mathbb{N} :]) \setminus \{\langle 0, 0\rangle\}$.

Then $\mathbb{N}$ is a subset of $\mathbb{R}$.

One can check the following observations:

* $\mathbb{C}$ is non empty,
* $\mathbb{Q}$ is non empty, and
* $\mathbb{Z}$ is non empty.

Next we state a number of propositions:

(1) $\mathbb{R} \subset \mathbb{C}$.

(2) $\mathbb{Q} \subset \mathbb{R}$.

(3) $\mathbb{Q} \subset \mathbb{C}$.

(4) $\mathbb{Z} \subset \mathbb{Q}$.

(5) $\mathbb{Z} \subset \mathbb{R}$.

(6) $\mathbb{Z} \subset \mathbb{C}$.

(7) $\mathbb{N} \subset \mathbb{Z}$.

(8) $\mathbb{N} \subset \mathbb{Q}$.

(9) $\mathbb{N} \subset \mathbb{R}$.

(10) $\mathbb{N} \subset \mathbb{C}$.

2. To Be Canceled

One can prove the following propositions:

(11) $\mathbb{R} \subseteq \mathbb{C}$.

(12) $\mathbb{Q} \subseteq \mathbb{R}$.

(13) $\mathbb{Q} \subseteq \mathbb{C}$.

(14) $\mathbb{Z} \subseteq \mathbb{Q}$.

(15) $\mathbb{Z} \subseteq \mathbb{R}$.

(16) $\mathbb{Z} \subseteq \mathbb{C}$.

(17) $\mathbb{N} \subseteq \mathbb{Z}$.

(18) $\mathbb{N} \subseteq \mathbb{Q}$.

(19) $\mathbb{N} \subseteq \mathbb{R}$.

(20) $\mathbb{N} \subseteq \mathbb{C}$.

(21) $\mathbb{R} \neq \mathbb{C}$.

(22) $\mathbb{Q} \neq \mathbb{R}$.

(23) $\mathbb{Q} \neq \mathbb{C}$.

(24) $\mathbb{Z} \neq \mathbb{Q}$.

(25) $\mathbb{Z} \neq \mathbb{R}$.

(26) $\mathbb{Z} \neq \mathbb{C}$.

(27) $\mathbb{N} \neq \mathbb{Z}$.

(28) $\mathbb{N} \neq \mathbb{Q}$.

(29) $\mathbb{N} \neq \mathbb{R}$.

(30) $\mathbb{N} \neq \mathbb{C}$.

References

[1] Grzegorz Bancerek. The ordinal numbers. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/ordinal1.html.

[2] Grzegorz Bancerek. Sequences of ordinal numbers. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/ordinal2.html.

[3] Grzegorz Bancerek. Arithmetic of non negative rational numbers. *Journal of Formalized Mathematics*, Addenda, 1998. http://mizar.org/JFM/Addenda/arytm_3.html.

[4] Czesław Byliński. Functions and their basic properties. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/funct_1.html.

[5] Czesław Byliński. Functions from a set to a set. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/funct_2.html.

[6] Czesław Byliński. Some basic properties of sets. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/zfmisc_1.html.

[7] Andrzej Trybulec. Tarski Grothendieck set theory. *Journal of Formalized Mathematics*, Axiomatics, 1989. http://mizar.org/JFM/Axiomatics/tarski.html.

[8] Andrzej Trybulec. Non negative real numbers. Part I. *Journal of Formalized Mathematics*, Addenda, 1998. http://mizar.org/JFM/Addenda/arytm_2.html.

[9] Zinaida Trybulec. Properties of subsets. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/subset_1.html.

[10] Edmund Woronowicz. Relations and their basic properties. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/relat_1.html.

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