JOURNAL OF FORMALIZED MATHEMATICS
Volume 14, Released 2002, Published 2003
Inst. of Computer Science, Univ. of Białystok

On the General Position of Special Polygons[1]

Mariusz Giero
University of Białystok

Summary. In this paper we introduce the notion of general position. We also show some auxiliary theorems for proving Jordan curve theorem. The following main theorems are proved:

1. End points of a polygon are in the same component of a complement of another polygon if number of common points of these polygons is even;
2. Two points of polygon L are in the same component of a complement of polygon M if two points of polygon M are in the same component of polygon L.

MML Identifier: `JORDAN12`.

WWW: `http://mizar.org/JFM/Vol14/jordan12.html`

The articles [22], [25], [19], [2], [17], [21], [15], [26], [1], [7], [5], [3], [24], [10], [4], [20], [18], [8], [9], [13], [14], [11], [12], [16], [23], and [6] provide the notation and terminology for this paper.

1. PRELIMINARIES

We adopt the following convention: i, j, k, n denote natural numbers, a, b, c, x denote sets, and r denotes a real number.

One can prove the following propositions:

(1) If $1 < i$, then $0 < i -' 1$.

(3)[1] 1 is odd.

(4) For every finite sequence f of elements of $\mathcal{E}^n_{\rm T}$ and for every i such that $1 \leq i$ and $i+1 \leq \operatorname{len} f$ holds $f_i \in \operatorname{rng} f$ and $f_{i+1} \in \operatorname{rng} f$.

One can verify that every finite sequence of elements of $\mathcal{E}^2_{\rm T}$ which is s.n.c. is also s.c.c..

One can prove the following two propositions:

(5) Let f, g be finite sequences of elements of $\mathcal{E}^2_{\rm T}$. If $f \frown g$ is unfolded and s.c.c. and $\operatorname{len} g \geq 2$, then f is unfolded and s.n.c..

(6) For all finite sequences g_1, g_2 of elements of $\mathcal{E}^2_{\rm T}$ holds $\widetilde{\mathcal{L}}(g_1) \subseteq \widetilde{\mathcal{L}}(g_1 \frown g_2)$.

[1]This work has been partially supported by CALCULEMUS grant HPRN-CT-2000-00102.

[1] The proposition (2) has been removed.

2. The Notion of General Position and Its Properties

Let us consider n and let f_1, f_2 be finite sequences of elements of $\mathcal{E}^n_{\mathrm{T}}$. We say that f_1 is in general position wrt f_2 if and only if:

(Def. 1) $\widetilde{\mathcal{L}}(f_1)$ misses $\operatorname{rng} f_2$ and for every i such that $1 \leq i$ and $i < \operatorname{len} f_2$ holds $\widetilde{\mathcal{L}}(f_1) \cap \mathcal{L}(f_2, i)$ is trivial.

Let us consider n and let f_1, f_2 be finite sequences of elements of $\mathcal{E}^n_{\mathrm{T}}$. We say that f_1 and f_2 are in general position if and only if:

(Def. 2) f_1 is in general position wrt f_2 and f_2 is in general position wrt f_1.

Let us note that the predicate f_1 and f_2 are in general position is symmetric.

We now state two propositions:

(7) Let f_1, f_2 be finite sequences of elements of $\mathcal{E}^2_{\mathrm{T}}$. Suppose f_1 and f_2 are in general position. Let f be a finite sequence of elements of $\mathcal{E}^2_{\mathrm{T}}$. If $f = f_2 \restriction \operatorname{Seg} k$, then f_1 and f are in general position.

(8) Let f_1, f_2, g_1, g_2 be finite sequences of elements of $\mathcal{E}^2_{\mathrm{T}}$. Suppose $f_1 \frown f_2$ and $g_1 \frown g_2$ are in general position. Then $f_1 \frown f_2$ and g_1 are in general position.

In the sequel f, g denote finite sequences of elements of $\mathcal{E}^2_{\mathrm{T}}$.

One can prove the following propositions:

(9) For all k, f, g such that $1 \leq k$ and $k+1 \leq \operatorname{len} g$ and f and g are in general position holds $g(k) \in (\widetilde{\mathcal{L}}(f))^{\mathrm{c}}$ and $g(k+1) \in (\widetilde{\mathcal{L}}(f))^{\mathrm{c}}$.

(10) Let f_1, f_2 be finite sequences of elements of $\mathcal{E}^2_{\mathrm{T}}$. Suppose f_1 and f_2 are in general position. Let given i, j. If $1 \leq i$ and $i+1 \leq \operatorname{len} f_1$ and $1 \leq j$ and $j+1 \leq \operatorname{len} f_2$, then $\mathcal{L}(f_1, i) \cap \mathcal{L}(f_2, j)$ is trivial.

(11) For all f, g holds $\{\mathcal{L}(f,i) : 1 \leq i \ \wedge \ i+1 \leq \operatorname{len} f\} \Cap \{\mathcal{L}(g,j) : 1 \leq j \ \wedge \ j+1 \leq \operatorname{len} g\}$ is finite.

(12) For all f, g such that f and g are in general position holds $\widetilde{\mathcal{L}}(f) \cap \widetilde{\mathcal{L}}(g)$ is finite.

(13) For all f, g such that f and g are in general position and for every k holds $\widetilde{\mathcal{L}}(f) \cap \mathcal{L}(g,k)$ is finite.

3. Properties of Being in the Same Component of a Complement of a Polygon

We use the following convention: f denotes a non constant standard special circular sequence and p, p_1, p_2, q denote points of $\mathcal{E}^2_{\mathrm{T}}$.

One can prove the following propositions:

(14) For all f, p_1, p_2 such that $\mathcal{L}(p_1, p_2)$ misses $\widetilde{\mathcal{L}}(f)$ there exists a subset C of $\mathcal{E}^2_{\mathrm{T}}$ such that C is a component of $(\widetilde{\mathcal{L}}(f))^{\mathrm{c}}$ and $p_1 \in C$ and $p_2 \in C$.

(15) There exists a subset C of $\mathcal{E}^2_{\mathrm{T}}$ such that C is a component of $(\widetilde{\mathcal{L}}(f))^{\mathrm{c}}$ and $a \in C$ and $b \in C$ if and only if $a \in \operatorname{RightComp}(f)$ and $b \in \operatorname{RightComp}(f)$ or $a \in \operatorname{LeftComp}(f)$ and $b \in \operatorname{LeftComp}(f)$.

(16) $a \in (\widetilde{\mathcal{L}}(f))^{\mathrm{c}}$ and $b \in (\widetilde{\mathcal{L}}(f))^{\mathrm{c}}$ and it is not true that there exists a subset C of $\mathcal{E}^2_{\mathrm{T}}$ such that C is a component of $(\widetilde{\mathcal{L}}(f))^{\mathrm{c}}$ and $a \in C$ and $b \in C$ if and only if $a \in \operatorname{LeftComp}(f)$ and $b \in \operatorname{RightComp}(f)$ or $a \in \operatorname{RightComp}(f)$ and $b \in \operatorname{LeftComp}(f)$.

(17) Let given f, a, b, c. Suppose that

(i) there exists a subset C of $\mathcal{E}^2_{\rm T}$ such that C is a component of $(\widetilde{\mathcal{L}}(f))^{\rm c}$ and $a \in C$ and $b \in C$, and

(ii) there exists a subset C of $\mathcal{E}^2_{\rm T}$ such that C is a component of $(\widetilde{\mathcal{L}}(f))^{\rm c}$ and $b \in C$ and $c \in C$.

Then there exists a subset C of $\mathcal{E}^2_{\rm T}$ such that C is a component of $(\widetilde{\mathcal{L}}(f))^{\rm c}$ and $a \in C$ and $c \in C$.

(18) Let given f, a, b, c. Suppose that

(i) $a \in (\widetilde{\mathcal{L}}(f))^{\rm c}$,

(ii) $b \in (\widetilde{\mathcal{L}}(f))^{\rm c}$,

(iii) $c \in (\widetilde{\mathcal{L}}(f))^{\rm c}$,

(iv) it is not true that there exists a subset C of $\mathcal{E}^2_{\rm T}$ such that C is a component of $(\widetilde{\mathcal{L}}(f))^{\rm c}$ and $a \in C$ and $b \in C$, and

(v) it is not true that there exists a subset C of $\mathcal{E}^2_{\rm T}$ such that C is a component of $(\widetilde{\mathcal{L}}(f))^{\rm c}$ and $b \in C$ and $c \in C$.

Then there exists a subset C of $\mathcal{E}^2_{\rm T}$ such that C is a component of $(\widetilde{\mathcal{L}}(f))^{\rm c}$ and $a \in C$ and $c \in C$.

4. Cells Are Convex

In the sequel G denotes a Go-board.

We now state several propositions:

(19) If $i \leq \operatorname{len} G$, then $\operatorname{vstrip}(G, i)$ is convex.

(20) If $j \leq \operatorname{width} G$, then $\operatorname{hstrip}(G, j)$ is convex.

(21) If $i \leq \operatorname{len} G$ and $j \leq \operatorname{width} G$, then $\operatorname{cell}(G, i, j)$ is convex.

(22) For all f, k such that $1 \leq k$ and $k+1 \leq \operatorname{len} f$ holds $\operatorname{leftcell}(f, k)$ is convex.

(23) For all f, k such that $1 \leq k$ and $k+1 \leq \operatorname{len} f$ holds left_cell(f, k, the Go-board of f) is convex and right_cell(f, k, the Go-board of f) is convex.

5. Properties of Points Lying on the Same Line

One can prove the following propositions:

(24) Let given p_1, p_2, f and r be a point of $\mathcal{E}^2_{\rm T}$. Suppose $r \in \mathcal{L}(p_1, p_2)$ and there exists x such that $\widetilde{\mathcal{L}}(f) \cap \mathcal{L}(p_1, p_2) = \{x\}$ and $r \notin \widetilde{\mathcal{L}}(f)$. Then $\widetilde{\mathcal{L}}(f)$ misses $\mathcal{L}(p_1, r)$ or $\widetilde{\mathcal{L}}(f)$ misses $\mathcal{L}(r, p_2)$.

(25) For all points p, q, r, s of $\mathcal{E}^2_{\rm T}$ such that $\mathcal{L}(p, q)$ is vertical and $\mathcal{L}(r, s)$ is vertical and $\mathcal{L}(p, q)$ meets $\mathcal{L}(r, s)$ holds $p_{\mathbf{1}} = r_{\mathbf{1}}$.

(26) For all p, p_1, p_2 such that $p \notin \mathcal{L}(p_1, p_2)$ and $(p_1)_{\mathbf{2}} = (p_2)_{\mathbf{2}}$ and $(p_2)_{\mathbf{2}} = p_{\mathbf{2}}$ holds $p_1 \in \mathcal{L}(p, p_2)$ or $p_2 \in \mathcal{L}(p, p_1)$.

(27) For all p, p_1, p_2 such that $p \notin \mathcal{L}(p_1, p_2)$ and $(p_1)_{\mathbf{1}} = (p_2)_{\mathbf{1}}$ and $(p_2)_{\mathbf{1}} = p_{\mathbf{1}}$ holds $p_1 \in \mathcal{L}(p, p_2)$ or $p_2 \in \mathcal{L}(p, p_1)$.

(28) If $p \neq p_1$ and $p \neq p_2$ and $p \in \mathcal{L}(p_1, p_2)$, then $p_1 \notin \mathcal{L}(p, p_2)$.

(29) Let given p, p_1, p_2, q. Suppose $q \notin \mathcal{L}(p_1, p_2)$ and $p \in \mathcal{L}(p_1, p_2)$ and $p \neq p_1$ and $p \neq p_2$ and $(p_1)_{\mathbf{1}} = (p_2)_{\mathbf{1}}$ and $(p_2)_{\mathbf{1}} = q_{\mathbf{1}}$ or $(p_1)_{\mathbf{2}} = (p_2)_{\mathbf{2}}$ and $(p_2)_{\mathbf{2}} = q_{\mathbf{2}}$. Then $p_1 \in \mathcal{L}(q, p)$ or $p_2 \in \mathcal{L}(q, p)$.

(30) Let p_1, p_2, p_3, p_4, p be points of $\mathcal{E}^2_{\rm T}$. Suppose $(p_1)_{\mathbf{1}} = (p_2)_{\mathbf{1}}$ and $(p_3)_{\mathbf{1}} = (p_4)_{\mathbf{1}}$ or $(p_1)_{\mathbf{2}} = (p_2)_{\mathbf{2}}$ and $(p_3)_{\mathbf{2}} = (p_4)_{\mathbf{2}}$ but $\mathcal{L}(p_1, p_2) \cap \mathcal{L}(p_3, p_4) = \{p\}$. Then $p = p_1$ or $p = p_2$ or $p = p_3$.

6. The Position of the Points of a Polygon with Respect to Another Polygon

Next we state several propositions:

(31) Let given p, p_1, p_2, f. Suppose $\widetilde{\mathcal{L}}(f) \cap \mathcal{L}(p_1, p_2) = \{p\}$. Let r be a point of $\mathcal{E}^2_{\rm T}$. Suppose that

(i) $r \notin \mathcal{L}(p_1, p_2)$,

(ii) $p_1 \notin \widetilde{\mathcal{L}}(f)$,

(iii) $p_2 \notin \widetilde{\mathcal{L}}(f)$,

(iv) $(p_1)_{\mathbf{1}} = (p_2)_{\mathbf{1}}$ and $(p_1)_{\mathbf{1}} = r_{\mathbf{1}}$ or $(p_1)_{\mathbf{2}} = (p_2)_{\mathbf{2}}$ and $(p_1)_{\mathbf{2}} = r_{\mathbf{2}}$,

(v) there exists i such that $1 \leq i$ and $i+1 \leq \operatorname{len} f$ and $r \in \text{right\_cell}(f, i, \text{the Go-board of } f)$ or $r \in \text{left\_cell}(f, i, \text{the Go-board of } f)$ and $p \in \mathcal{L}(f, i)$, and

(vi) $r \notin \widetilde{\mathcal{L}}(f)$.

Then

(vii) there exists a subset C of $\mathcal{E}^2_{\rm T}$ such that C is a component of $(\widetilde{\mathcal{L}}(f))^{\rm c}$ and $r \in C$ and $p_1 \in C$, or

(viii) there exists a subset C of $\mathcal{E}^2_{\rm T}$ such that C is a component of $(\widetilde{\mathcal{L}}(f))^{\rm c}$ and $r \in C$ and $p_2 \in C$.

(32) Let given f, p_1, p_2, p. Suppose $\widetilde{\mathcal{L}}(f) \cap \mathcal{L}(p_1, p_2) = \{p\}$. Let r_1, r_2 be points of $\mathcal{E}^2_{\rm T}$. Suppose that

(i) $p_1 \notin \widetilde{\mathcal{L}}(f)$,

(ii) $p_2 \notin \widetilde{\mathcal{L}}(f)$,

(iii) $(p_1)_{\mathbf{1}} = (p_2)_{\mathbf{1}}$ and $(p_1)_{\mathbf{1}} = (r_1)_{\mathbf{1}}$ and $(r_1)_{\mathbf{1}} = (r_2)_{\mathbf{1}}$ or $(p_1)_{\mathbf{2}} = (p_2)_{\mathbf{2}}$ and $(p_1)_{\mathbf{2}} = (r_1)_{\mathbf{2}}$ and $(r_1)_{\mathbf{2}} = (r_2)_{\mathbf{2}}$,

(iv) there exists i such that $1 \leq i$ and $i+1 \leq \operatorname{len} f$ and $r_1 \in \text{left\_cell}(f, i, \text{the Go-board of } f)$ and $r_2 \in \text{right\_cell}(f, i, \text{the Go-board of } f)$ and $p \in \mathcal{L}(f, i)$,

(v) $r_1 \notin \widetilde{\mathcal{L}}(f)$, and

(vi) $r_2 \notin \widetilde{\mathcal{L}}(f)$.

Then it is not true that there exists a subset C of $\mathcal{E}^2_{\rm T}$ such that C is a component of $(\widetilde{\mathcal{L}}(f))^{\rm c}$ and $p_1 \in C$ and $p_2 \in C$.

(33) Let given p, f, p_1, p_2. Suppose $\widetilde{\mathcal{L}}(f) \cap \mathcal{L}(p_1, p_2) = \{p\}$ and $(p_1)_{\mathbf{1}} = (p_2)_{\mathbf{1}}$ or $(p_1)_{\mathbf{2}} = (p_2)_{\mathbf{2}}$ and $p_1 \notin \widetilde{\mathcal{L}}(f)$ and $p_2 \notin \widetilde{\mathcal{L}}(f)$ and $\operatorname{rng} f$ misses $\mathcal{L}(p_1, p_2)$. Then it is not true that there exists a subset C of $\mathcal{E}^2_{\rm T}$ such that C is a component of $(\widetilde{\mathcal{L}}(f))^{\rm c}$ and $p_1 \in C$ and $p_2 \in C$.

(34) Let f be a non constant standard special circular sequence and g be a special finite sequence of elements of $\mathcal{E}^2_{\rm T}$. Suppose f and g are in general position. Let given k. Suppose $1 \leq k$ and $k+1 \leq \operatorname{len} g$. Then $\overline{\overline{\widetilde{\mathcal{L}}(f) \cap \mathcal{L}(g, k)}}$ is an even natural number if and only if there exists a subset C of $\mathcal{E}^2_{\rm T}$ such that C is a component of $(\widetilde{\mathcal{L}}(f))^{\rm c}$ and $g(k) \in C$ and $g(k+1) \in C$.

(35) Let f_1, f_2, g_1 be special finite sequences of elements of $\mathcal{E}^2_{\rm T}$. Suppose that

(i) $f_1 \frown f_2$ is a non constant standard special circular sequence,

(ii) $f_1 \frown f_2$ and g_1 are in general position,

(iii) $\operatorname{len} g_1 \geq 2$, and

(iv) g_1 is unfolded and s.n.c..

Then $\overline{\overline{\widetilde{\mathcal{L}}(f_1 \frown f_2) \cap \widetilde{\mathcal{L}}(g_1)}}$ is an even natural number if and only if there exists a subset C of $\mathcal{E}^2_{\rm T}$ such that C is a component of $(\widetilde{\mathcal{L}}(f_1 \frown f_2))^{\rm c}$ and $g_1(1) \in C$ and $g_1(\operatorname{len} g_1) \in C$.

(36) Let f_1, f_2, g_1, g_2 be special finite sequences of elements of $\mathcal{E}^2_{\mathrm{T}}$. Suppose that

(i) $f_1 \frown\!\frown f_2$ is a non constant standard special circular sequence,

(ii) $g_1 \frown\!\frown g_2$ is a non constant standard special circular sequence,

(iii) $\widetilde{\mathcal{L}}(f_1)$ misses $\widetilde{\mathcal{L}}(g_2)$,

(iv) $\widetilde{\mathcal{L}}(f_2)$ misses $\widetilde{\mathcal{L}}(g_1)$, and

(v) $f_1 \frown\!\frown f_2$ and $g_1 \frown\!\frown g_2$ are in general position.

Let p_1, p_2, q_1, q_2 be points of $\mathcal{E}^2_{\mathrm{T}}$. Suppose that $f_1(1) = p_1$ and $f_1(\mathrm{len}\, f_1) = p_2$ and $g_1(1) = q_1$ and $g_1(\mathrm{len}\, g_1) = q_2$ and $(f_1)_{\mathrm{len}\, f_1} = (f_2)_1$ and $(g_1)_{\mathrm{len}\, g_1} = (g_2)_1$ and $p_1 \in \widetilde{\mathcal{L}}(f_1) \cap \widetilde{\mathcal{L}}(f_2)$ and $q_1 \in \widetilde{\mathcal{L}}(g_1) \cap \widetilde{\mathcal{L}}(g_2)$ and there exists a subset C of $\mathcal{E}^2_{\mathrm{T}}$ such that C is a component of $(\widetilde{\mathcal{L}}(f_1 \frown\!\frown f_2))^{\mathrm{c}}$ and $q_1 \in C$ and $q_2 \in C$. Then there exists a subset C of $\mathcal{E}^2_{\mathrm{T}}$ such that C is a component of $(\widetilde{\mathcal{L}}(g_1 \frown\!\frown g_2))^{\mathrm{c}}$ and $p_1 \in C$ and $p_2 \in C$.

ACKNOWLEDGMENTS

I would like to thank Prof. Andrzej Trybulec for his help in preparation of this article. I also thank Adam Grabowski, Robert Milewski and Adam Naumowicz for their helpful comments.

REFERENCES

[1] Grzegorz Bancerek. Cardinal numbers. *Journal of Formalized Mathematics*, 1, 1989. `http://mizar.org/JFM/Vol1/card_1.html`.

[2] Grzegorz Bancerek. The fundamental properties of natural numbers. *Journal of Formalized Mathematics*, 1, 1989. `http://mizar.org/JFM/Vol1/nat_1.html`.

[3] Grzegorz Bancerek and Krzysztof Hryniewiecki. Segments of natural numbers and finite sequences. *Journal of Formalized Mathematics*, 1, 1989. `http://mizar.org/JFM/Vol1/finseq_1.html`.

[4] Józef Białas. Group and field definitions. *Journal of Formalized Mathematics*, 1, 1989. `http://mizar.org/JFM/Vol1/realset1.html`.

[5] Czesław Byliński. Functions and their basic properties. *Journal of Formalized Mathematics*, 1, 1989. `http://mizar.org/JFM/Vol1/funct_1.html`.

[6] Czesław Byliński. Some properties of cells on go board. *Journal of Formalized Mathematics*, 11, 1999. `http://mizar.org/JFM/Vol11/gobrd13.html`.

[7] Agata Darmochwał. Finite sets. *Journal of Formalized Mathematics*, 1, 1989. `http://mizar.org/JFM/Vol1/finset_1.html`.

[8] Agata Darmochwał. The Euclidean space. *Journal of Formalized Mathematics*, 3, 1991. `http://mizar.org/JFM/Vol3/euclid.html`.

[9] Agata Darmochwał and Yatsuka Nakamura. The topological space $\mathcal{E}^2_{\mathrm{T}}$. Arcs, line segments and special polygonal arcs. *Journal of Formalized Mathematics*, 3, 1991. `http://mizar.org/JFM/Vol3/topreal1.html`.

[10] Katarzyna Jankowska. Matrices. Abelian group of matrices. *Journal of Formalized Mathematics*, 3, 1991. `http://mizar.org/JFM/Vol3/matrix_1.html`.

[11] Jarosław Kotowicz and Yatsuka Nakamura. Introduction to Go-Board — part I. *Journal of Formalized Mathematics*, 4, 1992. `http://mizar.org/JFM/Vol4/goboard1.html`.

[12] Jarosław Kotowicz and Yatsuka Nakamura. Introduction to Go-Board — part II. *Journal of Formalized Mathematics*, 4, 1992. `http://mizar.org/JFM/Vol4/goboard2.html`.

[13] Yatsuka Nakamura and Czesław Byliński. Extremal properties of vertices on special polygons, part I. *Journal of Formalized Mathematics*, 6, 1994. `http://mizar.org/JFM/Vol6/sppol_1.html`.

[14] Yatsuka Nakamura and Jarosław Kotowicz. The Jordan's property for certain subsets of the plane. *Journal of Formalized Mathematics*, 4, 1992. `http://mizar.org/JFM/Vol4/jordan1.html`.

[15] Yatsuka Nakamura and Piotr Rudnicki. Vertex sequences induced by chains. *Journal of Formalized Mathematics*, 7, 1995. `http://mizar.org/JFM/Vol7/graph_2.html`.

[16] Yatsuka Nakamura and Andrzej Trybulec. Decomposing a Go-Board into cells. *Journal of Formalized Mathematics*, 7, 1995. `http://mizar.org/JFM/Vol7/goboard5.html`.

[17] Takaya Nishiyama and Yasuho Mizuhara. Binary arithmetics. *Journal of Formalized Mathematics*, 5, 1993. `http://mizar.org/JFM/Vol5/binarith.html`.

[18] Beata Padlewska. Connected spaces. *Journal of Formalized Mathematics*, 1, 1989. `http://mizar.org/JFM/Vol1/connsp_1.html`.

[19] Beata Padlewska. Families of sets. *Journal of Formalized Mathematics*, 1, 1989. `http://mizar.org/JFM/Vol1/setfam_1.html`.

[20] Beata Padlewska and Agata Darmochwał. Topological spaces and continuous functions. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/pre_topc.html.

[21] Piotr Rudnicki and Andrzej Trybulec. Abian's fixed point theorem. *Journal of Formalized Mathematics*, 9, 1997. http://mizar.org/JFM/Vol9/abian.html.

[22] Andrzej Trybulec. Tarski Grothendieck set theory. *Journal of Formalized Mathematics*, Axiomatics, 1989. http://mizar.org/JFM/Axiomatics/tarski.html.

[23] Andrzej Trybulec. Left and right component of the complement of a special closed curve. *Journal of Formalized Mathematics*, 7, 1995. http://mizar.org/JFM/Vol7/goboard9.html.

[24] Wojciech A. Trybulec. Pigeon hole principle. *Journal of Formalized Mathematics*, 2, 1990. http://mizar.org/JFM/Vol2/finseq_4.html.

[25] Zinaida Trybulec. Properties of subsets. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/subset_1.html.

[26] Edmund Woronowicz. Relations and their basic properties. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/relat_1.html.

Received May 27, 2002

Published January 2, 2004