JOURNAL OF FORMALIZED MATHEMATICS
Volume 7, Released 1995, Published 2003
Inst. of Computer Science, Univ. of Białystok

Left and Right Component of the Complement of a Special Closed Curve

Andrzej Trybulec
Warsaw University
Białystok

Summary. In the article the concept of the left and right component are introduced. These are the auxiliary notions needed in the proof of Jordan Curve Theorem.

MML Identifier: GOBOARD9.

WWW: http://mizar.org/JFM/Vol7/goboard9.html

The articles [18], [20], [3], [4], [2], [15], [6], [5], [10], [1], [17], [21], [16], [8], [9], [11], [12], [13], [7], [19], and [14] provide the notation and terminology for this paper.

For simplicity, we use the following convention: f is a non constant standard special circular sequence, i, j, k are natural numbers, a, b are natural numbers, p, q are points of $\mathcal{E}^2_{\mathrm{T}}$, and G is a Go-board.

One can prove the following propositions:

(1) $a -' a = 0$.

(2) $a -' b \leq a$.

(3) Let G_1 be a non empty topological space and A_1, A_2, B be subsets of G_1. Suppose A_1 is a component of B and A_2 is a component of B. Then $A_1 = A_2$ or A_1 misses A_2.

(4) Let G_1 be a non empty topological space, A, B be non empty subsets of G_1, and A_3 be a subset of $G_1 \restriction B$. If $A = A_3$, then $G_1 \restriction A = G_1 \restriction B \restriction A_3$.

(5) Let G_1 be a non empty topological space, A be a non empty subset of G_1, and B be a non empty subset of G_1. Suppose $A \subseteq B$ and A is connected. Then there exists a subset C of G_1 such that C is a component of B and $A \subseteq C$.

(6) Let G_1 be a non empty topological space, A, C, D be subsets of G_1, and B be a subset of G_1. Suppose B is connected and C is a component of D and $A \subseteq C$ and A meets B and $B \subseteq D$. Then $B \subseteq C$.

(7) $\mathcal{L}(p,q)$ is convex.

(8) $\mathcal{L}(p,q)$ is connected.

Let us note that there exists a subset of $\mathcal{E}^2_{\mathrm{T}}$ which is convex and non empty.

One can prove the following propositions:

(9) For all convex subsets P, Q of $\mathcal{E}^2_{\mathrm{T}}$ holds $P \cap Q$ is convex.

(10) For every finite sequence f of elements of $\mathcal{E}^2_{\rm T}$ holds $\mathrm{Rev}(\mathbf{X}\text{-coordinate}(f)) = \mathbf{X}\text{-coordinate}(\mathrm{Rev}(f))$.

(11) For every finite sequence f of elements of $\mathcal{E}^2_{\rm T}$ holds $\mathrm{Rev}(\mathbf{Y}\text{-coordinate}(f)) = \mathbf{Y}\text{-coordinate}(\mathrm{Rev}(f))$.

Let us mention that there exists a finite sequence which is non constant.

Let f be a non constant finite sequence. One can verify that $\mathrm{Rev}(f)$ is non constant.

Let f be a standard special circular sequence. Then $\mathrm{Rev}(f)$ is a standard special circular sequence.

One can prove the following propositions:

(12) If $i \geq 1$ and $j \geq 1$ and $i+j = \mathrm{len}\, f$, then $\mathrm{leftcell}(f,i) = \mathrm{rightcell}(\mathrm{Rev}(f), j)$.

(13) If $i \geq 1$ and $j \geq 1$ and $i+j = \mathrm{len}\, f$, then $\mathrm{leftcell}(\mathrm{Rev}(f),i) = \mathrm{rightcell}(f, j)$.

(14) Suppose $1 \leq k$ and $k+1 \leq \mathrm{len}\, f$. Then there exist i, j such that $i \leq \mathrm{len}$ the Go-board of f and $j \leq \mathrm{width}$ the Go-board of f and cell(the Go-board of f, $i, j) = \mathrm{leftcell}(f,k)$.

(15) If $j \leq \mathrm{width}\, G$, then $\mathrm{Int}\,\mathrm{hstrip}(G, j)$ is convex.

(16) If $i \leq \mathrm{len}\, G$, then $\mathrm{Int}\,\mathrm{vstrip}(G,i)$ is convex.

(17) If $i \leq \mathrm{len}\, G$ and $j \leq \mathrm{width}\, G$, then $\mathrm{Int}\,\mathrm{cell}(G,i, j) \neq \emptyset$.

(18) If $1 \leq k$ and $k+1 \leq \mathrm{len}\, f$, then $\mathrm{Int}\,\mathrm{leftcell}(f,k) \neq \emptyset$.

(19) If $1 \leq k$ and $k+1 \leq \mathrm{len}\, f$, then $\mathrm{Int}\,\mathrm{rightcell}(f,k) \neq \emptyset$.

(20) If $i \leq \mathrm{len}\, G$ and $j \leq \mathrm{width}\, G$, then $\mathrm{Int}\,\mathrm{cell}(G,i, j)$ is convex.

(21) If $i \leq \mathrm{len}\, G$ and $j \leq \mathrm{width}\, G$, then $\mathrm{Int}\,\mathrm{cell}(G,i, j)$ is connected.

(22) If $1 \leq k$ and $k+1 \leq \mathrm{len}\, f$, then $\mathrm{Int}\,\mathrm{leftcell}(f,k)$ is connected.

(23) If $1 \leq k$ and $k+1 \leq \mathrm{len}\, f$, then $\mathrm{Int}\,\mathrm{rightcell}(f,k)$ is connected.

Let us consider f. The functor $\mathrm{LeftComp}(f)$ yields a subset of $\mathcal{E}^2_{\rm T}$ and is defined by:

(Def. 1) $\mathrm{LeftComp}(f)$ is a component of $(\widetilde{\mathcal{L}}(f))^{\rm c}$ and $\mathrm{Int}\,\mathrm{leftcell}(f,1) \subseteq \mathrm{LeftComp}(f)$.

The functor $\mathrm{RightComp}(f)$ yielding a subset of $\mathcal{E}^2_{\rm T}$ is defined as follows:

(Def. 2) $\mathrm{RightComp}(f)$ is a component of $(\widetilde{\mathcal{L}}(f))^{\rm c}$ and $\mathrm{Int}\,\mathrm{rightcell}(f,1) \subseteq \mathrm{RightComp}(f)$.

The following propositions are true:

(24) For every k such that $1 \leq k$ and $k+1 \leq \mathrm{len}\, f$ holds $\mathrm{Int}\,\mathrm{leftcell}(f,k) \subseteq \mathrm{LeftComp}(f)$.

(25) The Go-board of $\mathrm{Rev}(f) =$ the Go-board of f.

(26) $\mathrm{RightComp}(f) = \mathrm{LeftComp}(\mathrm{Rev}(f))$.

(27) $\mathrm{RightComp}(\mathrm{Rev}(f)) = \mathrm{LeftComp}(f)$.

(28) For every k such that $1 \leq k$ and $k+1 \leq \mathrm{len}\, f$ holds $\mathrm{Int}\,\mathrm{rightcell}(f,k) \subseteq \mathrm{RightComp}(f)$.

REFERENCES

[1] Grzegorz Bancerek. Cardinal numbers. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/card_1.html.

[2] Grzegorz Bancerek. The fundamental properties of natural numbers. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/nat_1.html.

[3] Grzegorz Bancerek. The ordinal numbers. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/ordinal1.html.

[4] Grzegorz Bancerek. Sequences of ordinal numbers. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/ordinal2.html.

[5] Grzegorz Bancerek and Krzysztof Hryniewiecki. Segments of natural numbers and finite sequences. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/finseq_1.html.

[6] Czesław Byliński. Functions and their basic properties. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/funct_1.html.

[7] Czesław Byliński. Some properties of restrictions of finite sequences. *Journal of Formalized Mathematics*, 7, 1995. http://mizar.org/JFM/Vol7/finseq_5.html.

[8] Agata Darmochwał. The Euclidean space. *Journal of Formalized Mathematics*, 3, 1991. http://mizar.org/JFM/Vol3/euclid.html.

[9] Agata Darmochwał and Yatsuka Nakamura. The topological space $\mathcal{E}^2_{\rm T}$. Arcs, line segments and special polygonal arcs. *Journal of Formalized Mathematics*, 3, 1991. http://mizar.org/JFM/Vol3/topreal1.html.

[10] Katarzyna Jankowska. Matrices. Abelian group of matrices. *Journal of Formalized Mathematics*, 3, 1991. http://mizar.org/JFM/Vol3/matrix_1.html.

[11] Jarosław Kotowicz and Yatsuka Nakamura. Introduction to Go-Board — part I. *Journal of Formalized Mathematics*, 4, 1992. http://mizar.org/JFM/Vol4/goboard1.html.

[12] Jarosław Kotowicz and Yatsuka Nakamura. Introduction to Go-Board — part II. *Journal of Formalized Mathematics*, 4, 1992. http://mizar.org/JFM/Vol4/goboard2.html.

[13] Yatsuka Nakamura and Jarosław Kotowicz. The Jordan's property for certain subsets of the plane. *Journal of Formalized Mathematics*, 4, 1992. http://mizar.org/JFM/Vol4/jordan1.html.

[14] Yatsuka Nakamura and Andrzej Trybulec. Decomposing a Go-Board into cells. *Journal of Formalized Mathematics*, 7, 1995. http://mizar.org/JFM/Vol7/goboard5.html.

[15] Takaya Nishiyama and Yasuho Mizuhara. Binary arithmetics. *Journal of Formalized Mathematics*, 5, 1993. http://mizar.org/JFM/Vol5/binarith.html.

[16] Beata Padlewska. Connected spaces. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/connsp_1.html.

[17] Beata Padlewska and Agata Darmochwał. Topological spaces and continuous functions. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/pre_topc.html.

[18] Andrzej Trybulec. Tarski Grothendieck set theory. *Journal of Formalized Mathematics*, Axiomatics, 1989. http://mizar.org/JFM/Axiomatics/tarski.html.

[19] Andrzej Trybulec. On the decomposition of finite sequences. *Journal of Formalized Mathematics*, 7, 1995. http://mizar.org/JFM/Vol7/finseq_6.html.

[20] Zinaida Trybulec. Properties of subsets. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/subset_1.html.

[21] Mirosław Wysocki and Agata Darmochwał. Subsets of topological spaces. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/tops_1.html.

Received October 29, 1995

Published January 2, 2004